Finite Causal-Screen Holography:
Boundary Capacity, Bulk Recovery, and Gluing Obstruction

Yining Wu
Independent Researcher
`yining.wu@alumni.upenn.edu`

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This paper introduces finite causal-screen holography as the first holographic bridge of the Finite Distinction Systems program. The central claim is that holography should be understood first as finite boundary recovery: a bulk distinction is physically accessible relative to a screen only when it can be encoded, maintained, and recovered through a finite boundary ledger. Standard AdS/CFT and related asymptotic holographic structures are treated as sharp high-capacity model classes in which recovery deficits and gluing obstructions are idealized away or controlled. In this framework, holographic entropy is interpreted as a screen-capacity ledger, entanglement-wedge reconstruction as finite boundary distinction recovery, and reconstruction failure as capacity deficit, leakage, or gluing obstruction. Finite causal diamonds and soft gravitational edge modes supply candidate screen variables for a later connection to G1 screen-response geometry.

This paper does not derive the G1 residual branch, the M3/4 cosmological channel, the 3/4 Weyl projection, rank-one horizon memory, full de Sitter holography, or microscopic quantum gravity. Its purpose is narrower: to define the finite-screen holographic objects needed for the H-series. The follow-up H2 paper will address recovery holonomy and its relation to the screen-response obstruction $d\omega$. The follow-up H3 paper will address whether finite-screen recovery can constrain the M3/4 branch. The compressed thesis is

$$\boxed{\text{holography is finite boundary distinction recovery under screen-capacity constraints.}}$$

Scope and Claim Status. H1 formulates holography as a finite recovery problem. Its central claim is that a bulk distinction is physically accessible relative to a finite screen only when a boundary ledger supplies enough capacity, redundancy, recovery maps, and gluing consistency to reconstruct it at the declared tolerance. H1 therefore defines finite screen ledgers, screen-relative bulk distinctions, recovery nets, exact and tolerant gluing criteria, and recovery-obstruction diagnostics.

This is not a claim of a new exact holographic duality and not a derivation of AdS/CFT, de Sitter holography, black-hole microstates, microscopic quantum gravity, G1/M3/4, the 3/4 Weyl projection, or rank-one horizon memory. H1 is the first H-series bridge paper: H2 will test whether recovery obstruction projects to screen-response curl, while H3 will test whether restricted optical-screen recovery can constrain the $M_{3/4}$ branch.

Claim-status summary

Keywords: finite distinction systems; holography; finite causal screens; causal diamonds; boundary capacity; bulk reconstruction; entanglement wedge; holographic quantum error correction; soft gravity; screen entropy; G1; recovery holonomy.

INTRODUCTION: THE FINITE-SCREEN TURN

From ideal boundaries to finite screens

Holography is often introduced through ideal boundary structures. In the most familiar form,

$$\text{bulk gravity} \quad \longleftrightarrow \quad \text{boundary quantum theory,} \tag{1}$$

with AdS/CFT providing the sharpest realization [1–3]. This idealized form is powerful because the asymptotic boundary is clean, the boundary theory is sharply defined, and the bulk reconstruction problem can be studied with exact tools. The holographic principle [22] and the covariant entropy bound [21] provide the general background that boundary area limits physical degrees of freedom; H1 translates this into finite screen-capacity language.

Finite physical observers, however, do not generally possess ideal asymptotic boundaries. They operate through finite causal domains, finite screens, finite detectors, finite update windows, and finite recovery tolerances. A cosmological horizon, a finite causal diamond, a subregion boundary, and a laboratory screen are not merely approximate versions of an asymptotic boundary. They are finite ledgers of accessible distinction. The finite screen is not just a surface in a pre-given geometry; it is the boundary through which a finite system can encode, maintain, compare, and recover physical differences.

H1 therefore begins with a finite-screen question rather

TABLE I. Central FDS-H1 claims, status, and demotion conditions.

Claim	**Status**	**Demotion or limitation**
Bulk distinctions are screen-physical only when recoverable from a finite boundary ledger	FDS-H primitive / definition	Demotes if bulk recoverability cannot be formulated as a finite boundary-capacity and recovery problem without losing operational meaning
Finite causal-diamond edge variables can serve as candidate screen variables	Bridge principle	Demotes if finite causal-diamond variables cannot be related to recoverable boundary distinctions or response variables
Recovery nets provide finite gluing diagnostics for bulk reconstruction	Restricted theorem — exact and tolerant finite recovery nets	Demotes if path-dependent finite recovery cannot be captured by recovery transports, loop mismatch, or gluing obstruction
RT/FLM generalized entropy is a high-capacity realization of screen-capacity accounting	Bridge principle, not derivation	Demotes to heuristic dictionary if generalized entropy cannot be read as recovery-capacity accounting while preserving reconstruction content
Holographic QEC is finite boundary distinction recovery	Operational bridge	Demotes if reconstruction failure in controlled finite models is not captured by capacity deficit, leakage, or boundary redundancy structure
H1 prepares, but does not prove, the H2 bridge from recovery holonomy to $d\omega$	Open handoff	H1 remains a finite-screen architecture if H2 fails; H2 failure demotes the H-to-G1 derivation route, not the H1 recovery vocabulary

than an asymptotic-duality question:

> *What finite boundary ledger makes a bulk distinction recoverable?*

This reframes holography as a finite recovery problem. The point is not to reject AdS/CFT, RT/HRT, FLM, islands, or holographic quantum error correction. The point is to reclassify them as sharp realizations of a broader finite-boundary recovery architecture.

The FDS question

The Finite Distinction Systems program treats a physical distinction as finite, boundary-relative, and maintained. A distinction is not physically available merely because a formal state space contains different labels. It is physical relative to a boundary when it can be accessed, preserved, recovered, recorded, or otherwise maintained under finite capacity and finite error.

In the holographic setting, this means that a bulk distinction Q_{bulk} is screen-physical relative to a boundary Σ only when boundary data on Σ support an admissible recovery map. Thus H1 replaces the slogan "bulk equals boundary" with the finite statement

$$\boxed{\text{bulk accessibility is finite boundary recoverability.}} \tag{2}$$

The boundary is not an abstract duplicate of the bulk. It is a finite ledger with limited capacity, redundancy, error tolerance, and recovery structure.

Three shifts

H1 makes three shifts.

First, it shifts from boundary theory to boundary ledger. A boundary ledger includes not only a Hilbert space or algebra but also capacity, record structure, redundancy, recovery maps, and finite task tolerances.

Second, it shifts from formal bulk existence to recoverable bulk distinction. A bulk operator, region, or field difference becomes physically accessible relative to a screen only when it lies inside the recoverable distinction sector of that screen.

Third, it shifts from asymptotic holography to finite causal-screen holography. AdS/CFT remains a sharp ideal model class, but finite causal screens are the primary FDS object because they are the natural boundary objects for finite observers and finite gravitational response.

Relation to G1

FDS-G1 begins with finite causal-screen entropy response:

$$S_{\mathrm{scr}} = k_B \log N_{\mathrm{scr}}, \qquad \omega = \eta_i(X)\,\mathrm{d}X^i. \tag{3}$$

The closed branch is

$$d\omega = 0 \quad \Rightarrow \quad \text{integrable metric envelope,} \tag{4}$$

while the nonclosed branch is

$$d\omega \neq 0 \quad \Rightarrow \quad \text{screen residual sector.} \tag{5}$$

H1 does not prove these G1 claims. It supplies candidate holographic origins for the objects appearing in them: finite screen variables X^i, finite screen capacity $N_{\rm scr}$, recovery consistency conditions, and a possible future route from finite recovery obstruction to $d\omega$. The bridge from recovery holonomy to screen-response curl is deliberately postponed to H2. The M3/4 branch is postponed to H3.

FINITE BOUNDARY RECOVERY PRIMITIVE

Definition 1 (Finite screen ledger)**.** *A finite screen ledger is a tuple*

$$\mathcal{L}_\Sigma = (\Sigma, \mathcal{B}_\Sigma, C_\Sigma, S_\Sigma, \mathcal{A}_\Sigma, \mathcal{R}_\Sigma), \tag{6}$$

where Σ is a finite boundary or causal screen, $\mathcal{B}_\Sigma$ is the accessible boundary data algebra or record set, C_Σ is the finite boundary capacity, S_Σ is the entropy ledger, $\mathcal{A}_\Sigma$ is the encoding or access map from bulk distinctions to boundary data, and $\mathcal{R}_\Sigma$ is the recovery map from boundary data to bulk distinctions.

The term "ledger" is intentional. A screen does not merely store labels. It accounts for which distinctions can be encoded, stabilized, compared, transported, erased, or recovered. The ledger includes capacity constraints and recovery constraints, not just kinematical boundary degrees of freedom.

Definition 2 (Screen-relative bulk distinction)**.** *A bulk distinction $Q_{\rm bulk}$ is screen-physical relative to Σ at tolerance $\epsilon_{\rm task}$ if*

$$\epsilon_\Sigma(Q) = \inf_{\mathcal{R}_\Sigma} D(Q_{\rm bulk}, \mathcal{R}_\Sigma(\mathcal{B}_\Sigma)) \le \epsilon_{\rm task}, \tag{7}$$

where D is an operational distinguishability measure appropriate to the task.

This definition is deliberately operational. It does not require a universal metric on all possible bulk descriptions. The correct D depends on the recovery task: operator reconstruction, state reconstruction, correlation recovery, causal distinguishability, or code-subspace fidelity.

Definition 3 (Screen capacity)**.** *The finite distinction capacity of a screen is measured in bits:*

$$C_\Sigma \equiv I_\Sigma = \log_2 N_\Sigma, \tag{8}$$

where N_Σ is the maximum number of mutually distinguishable screen states or screen-equivalence classes accessible under the declared boundary conditions. Throughout this paper, C_Σ and I_Σ are used interchangeably for the information-capacity ledger; the entropy form is $S_\Sigma = k_B \ln 2 \cdot I_\Sigma$. H1 does not claim to derive the Bekenstein-Hawking coefficient. This is the FDS finite ledger definition. The relation between this ledger and the gravitational area coefficient is part of the G1/D0 and later H-series problem.

Definition 4 (Recovery demand)**.** *The minimal recovery demand $R^{\mathcal{T}}_{\rm min}(Q, \epsilon, \tau)$ is the minimal boundary information required to recover a bulk distinction Q within error ϵ and time or update window τ under a declared recovery task $\mathcal{T}$. The task type $\mathcal{T}$ selects among operator reconstruction, state reconstruction, correlation recovery, causal distinguishability, or code-subspace fidelity.*

Definition 5 (Capacity deficit)**.** *The screen-relative capacity deficit is*

$$\Delta^{\mathcal{T}}_\Sigma(Q) = R^{\mathcal{T}}_{\rm min}(Q, \epsilon, \tau) - C^{\rm eff}_\Sigma. \tag{9}$$

If $\Delta_\Sigma(Q) > 0$, then Q is not recoverable from Σ without externalization, enlarged boundary access, additional redundancy, hidden side records, or relaxed task tolerance.

Criterion 1 (Screen-physicality criterion)**.** *A bulk distinction is physically accessible relative to a finite screen only if it lies in the recoverable sector*

$$\mathcal{Q}_{\rm rec}(\Sigma) = \{Q : \epsilon_\Sigma(Q) \le \epsilon_{\rm task}\}. \tag{10}$$

The complement is not denied as formal bulk structure; it is simply not screen-physical relative to Σ under the declared finite boundary conditions.

Example: operator-algebra recovery task. In holographic QEC, the task $\mathcal{T}$ selects operator reconstruction within a code subspace $\mathcal{H}_{\rm code}$. For a bulk operator O_a and a boundary operator O_A supported on region A, the recovery error is

$$\epsilon^{\rm op}_A(O_a) = \inf_{O_A} \sup_{\|\psi\|=1,\ \psi\in\mathcal{H}_{\rm code}} \|(O_A - O_a)|\psi\rangle\|. \tag{11}$$

The condition $\epsilon^{\rm op}_A(O_a) \le \epsilon_{\rm task}$ is a concrete instance of H1's screen-relative recoverability criterion. The norm may be replaced by trace-distance, diamond-norm, or correlation-loss variants depending on the recovery task $\mathcal{T}$.

FINITE CAUSAL SCREENS AND EDGE VARIABLES

Causal screen as boundary ledger

A finite causal screen is not merely a geometric surface. In H1 it is a finite ledger of recoverable causal distinctions. A screen can be a causal diamond boundary, a horizon patch, a subregion boundary, or an operationally defined observer screen. What makes it a screen is not its ideal location at infinity but its role as a finite boundary through which causal distinctions become accessible.

This shift is especially important because several recent gravitational programs connect asymptotic soft data, causal diamonds, and edge degrees of freedom. In particular, the phase-space relation between asymptotically flat soft gravity and finite causal diamonds suggests that finite screens are not merely heuristic cutoffs but candidate carriers of gravitational boundary data [15]. Recent finite-causal-diamond thermodynamic analyses further support the view that causal diamonds carry intrinsic entropy, modular, and geometric fluctuation data [23]. H1 does not import that result as a proof of G1. It uses it as external motivation for treating finite causal-screen variables as legitimate holographic data carriers.

Candidate screen variables

A finite causal screen may carry variables of the form

$$X^i \in \big\{\delta L,\ \delta A,\ \Pi_L,\ \text{shape modes}, \\ \text{soft edge modes},\ \text{memory modes}\big\}. \tag{12}$$

Here δL is a size or radial fluctuation, δA is an area fluctuation, Π_L is a conjugate response variable, shape modes encode nonspherical boundary deformation, soft edge modes encode low-energy gravitational boundary data, and memory modes encode persistent finite-screen response history.

This list is not claimed to be complete. It is a candidate set for finite-screen holography. Later H-series work may refine it into a specific screen phase space.

FDS translation

The FDS translation is

$$\begin{aligned} \text{finite causal-diamond phase space} &\longrightarrow \text{finite screen variables} \\ &\longrightarrow \text{finite screen ledger}. \end{aligned} \tag{13}$$

The critical step is not merely assigning variables to a boundary. It is assigning finite recovery meaning to those variables: which bulk distinctions do they make recoverable, at what capacity cost, under what tolerance, and with what gluing consistency across screen subregions?

Bridge Principle 1 (Finite causal-diamond variables as screen carriers)**.** *Finite causal-diamond edge variables are candidate carriers of finite screen distinctions.*

Remark 1 (Status of the finite-diamond bridge)**.** *This is a bridge principle, not a completed theorem. It demotes if finite causal-diamond variables cannot be related to recoverable boundary distinctions, capacity ledgers, or screen response variables.*

RECOVERY NETS AND GLUING OBSTRUCTION

Definition 6 (Finite recovery net)**.** *A finite recovery net* $\mathcal{R}$ *consists of:*

- ***Components:*** *finite boundary regions* $A_i \subseteq \Sigma$*, boundary records* B_{A_i}*, and recoverable bulk distinction sectors* $Q(A_i)$*;*
- ***Maps:*** *local recovery maps* $R_{A_i} : B_{A_i} \to Q(A_i)$*, restriction maps* $\rho_{A_iA_j} : B_{A_i} \to B_{A_j}$ *for* $A_j \subseteq A_i$*, transition maps* $T_{ij} : Q(A_i) \to Q(A_j)$ *on overlaps, and coarse-graining maps for larger regions;*
- ***Distinguishability structure:*** *a functional* $D_Q(\cdot,\cdot)$ *on recovered bulk distinctions, such as total variation, trace distance, or task loss.*

A recovery net is tolerance-enriched when each transition map carries a per-edge tolerance bound ϵ_{ij} *such that* $\|T_{ij}Q - Q\|_D \le \epsilon_{ij}$*.*

Boundary covers

Let a finite screen Σ be covered by boundary regions

$$\mathcal{U} = \{A_i\}_{i\in I}, \qquad A_i \subseteq \Sigma. \tag{14}$$

Each region has its own boundary data $\mathcal{B}_{A_i}$, capacity C_{A_i}, and recoverable sector.

Definition 7 (Local recoverable sector)**.** *The local recoverable sector associated with* A_i *is*

$$\mathcal{Q}(A_i) = \{Q_{\text{bulk}} : \epsilon_{A_i}(Q) \le \epsilon_{\text{task}}\}. \tag{15}$$

Definition 8 (Local recovery map)**.** *A local recovery map is*

$$\mathcal{R}_{A_i} : \mathcal{B}_{A_i} \to \mathcal{Q}(A_i). \tag{16}$$

It assigns to admissible boundary data a recovered bulk distinction, state, operator, correlation pattern, or equivalence class.

Overlap consistency

For overlapping regions $A_i \cap A_j \neq \emptyset$, local recoveries are consistent on the overlap if

$$D\big(\mathcal{R}_{A_i}(B_{A_i}), \mathcal{R}_{A_j}(B_{A_j})\big) \le \epsilon_{\text{glue}} \tag{17}$$

for all bulk distinctions in the common recoverable sector and for the declared gluing tolerance ϵ_{glue}.

This condition is a finite analogue of gluing local descriptions into a coherent global reconstruction. The key point is that finite recovery may be locally possible but globally inconsistent.

Recovery transport

Definition 9 (Recovery transport). *A recovery transport map between local recovery frames is*

$$T_{ij} : \mathcal{Q}(A_i) \to \mathcal{Q}(A_j), \tag{18}$$

where T_{ij} compares the same recovered bulk distinction across the recovery frames associated with A_i and A_j.

The transport map may be induced by overlap restriction, code-subspace isomorphism, entanglement-wedge inclusion, tensor-network recovery transition, or operational comparison between recovered observables. Related modular-flow reconstruction frameworks have been studied in [25].

Recovery loop and holonomy

For a loop of boundary recovery frames

$$A_i \to A_j \to A_k \to A_i, \tag{19}$$

define the recovery holonomy

$$\Omega_{ijk} = T_{ki}T_{jk}T_{ij} - \mathrm{id}. \tag{20}$$

More generally, for a loop $\gamma : A_1 \to A_2 \to \cdots \to A_n \to A_1$,

$$\Omega_{\mathrm{rec}}(\gamma) = T_{n1}T_{n-1,n}\cdots T_{12} - \mathrm{id}. \tag{21}$$

If $\Omega_{\mathrm{rec}} = 0$, finite recovery glues consistently around the loop. If $\Omega_{\mathrm{rec}} \neq 0$, the same bulk distinction cannot be recovered consistently around the finite boundary loop. This is a finite-screen gluing obstruction.

Theorem 1 (Finite recovery-net theorem — exact (nerve-graph) case). *Let $\mathcal{U} = \{A_1, \ldots, A_n\}$ be a finite cover of a bulk region and let $G_{\mathcal{U}}$ be its nerve graph: vertices are cover elements, edges join overlapping regions. On each connected component of $G_{\mathcal{U}}$, local recoverable sectors $Q(A_i)$ carry transition maps $T_{ij} : Q(A_i) \to Q(A_j)$ with $T_{ii} = \mathrm{id}$ and $T_{ji} = T_{ij}^{-1}$. Path-independent recovery of Q holds for all initial regions on that component if and only if for every cycle γ on the nerve graph, the ordered holonomy $T_\gamma = \mathrm{id}$.*

Proof. Path independence means that for any two paths p, q from A_i to A_j, the composed transport satisfies $T_p = T_q$. Equivalently, $T_p^{-1}T_q = \mathrm{id}$ for the loop formed by concatenating p^{-1} and q. It suffices to check trivial holonomy on a cycle basis of the nerve graph (equivalently, on all cycles). Conversely, if all cycle holonomies vanish, any two recovery paths produce identical bulk distinction assignments and recovery is path-independent. □

Theorem 2 (Finite recovery-net theorem — tolerant case). *Let a distinguishability distance $D_Q(\cdot,\cdot)$ be defined on recovered bulk distinctions, and let each transition map satisfy a Lipschitz condition*

$$D_Q(T_{ij}x,\, T_{ij}y) \leq L_{ij}\, D_Q(x, y) \tag{22}$$

with per-edge error bound $\|T_{ij}Q - Q\|_D \leq \epsilon_{ij}$. Then the recovery error along any path γ accumulates as

$$\epsilon_{\mathrm{path}} \leq \sum_{r=1}^{m}\Big(\prod_{s=r+1}^{m} L_s\Big)\,\epsilon_r, \tag{23}$$

where L_s and ϵ_r are indexed along the path. The total loop holonomy is bounded by the same expression.

Corollary 1 (1-Lipschitz / non-expansive recovery). *If all transition maps are non-expansive with respect to D_Q ($L_{ij} \leq 1$ for all i, j), the path error reduces to the simple sum*

$$\epsilon_{\mathrm{path}} \leq \sum_{(ij)\in\gamma} \epsilon_{ij}, \tag{24}$$

and the loop holonomy satisfies

$$\|T_\gamma Q - Q\|_D \leq \sum_{(ij)\in\gamma} \epsilon_{ij}. \tag{25}$$

Proof of Theorem 2. Let E_r be the deviation from the ideal transported recovery after r transitions, with $E_0 = 0$ and the initial per-edge bound

$$E_1 \leq \epsilon_{i_1 i_2}. \tag{26}$$

For each subsequent step, the Lipschitz property and the per-edge error bound give the recurrence

$$E_{r+1} \leq L_{i_{r+1}i_{r+2}}\, E_r + \epsilon_{i_{r+1}i_{r+2}}. \tag{27}$$

Unrolling the recurrence yields the product-weighted sum of Theorem 2. The loop bound follows as the special case where γ returns to the starting region. □

Criterion 2 (Demotion of the recovery-net formalism). *If recovery consistency cannot be captured by loop holonomy, gluing obstruction, or finite overlap consistency in controlled finite models, the recovery-net formalism demotes to a heuristic organizational language.*

SCREEN CAPACITY AND RESPONSE

Screen entropy ledger

The finite screen ledger records accessible distinction classes. We use dual notation to separate the counting (bits) from thermodynamic (entropy) conventions:

$$I_\Sigma = \log_2 N_{\rm scr}, \qquad S_{\rm scr} = k_B \ln 2 \cdot I_\Sigma = k_B \ln N_{\rm scr}. \quad (28)$$

Here I_Σ is the information capacity in bits and $S_{\rm scr}$ is the corresponding entropy. Throughout, capacity accounting uses the bit ledger I_Σ; the entropy form $S_{\rm scr}$ is retained where thermodynamic bridge objects (RT/FLM, Bekenstein-Hawking) are referenced. This definition is not yet a derivation of horizon entropy or Newton's constant. It is the FDS finite-capacity object from which later G1 and H-series work may attempt to derive gravitational response.

Capacity potential

Define a dimensionless capacity potential

$$\Phi(X) = \frac{S_{\rm scr}(X)}{k_B}. \quad (29)$$

In an ideal exact response regime, changes in screen capacity are generated by

$$\omega = d\Phi. \quad (30)$$

Response one-form

Following the G1 notation, define the screen response one-form

$$\omega = \eta_i(X)\, dX^i. \quad (31)$$

In the exact branch,

$$\omega = d\Phi, \qquad d\omega = 0. \quad (32)$$

Residual response component

Finite recovery may fail to glue exactly. H1 therefore introduces the decomposition

$$\omega = d\Phi + \alpha, \quad (33)$$

where α is a residual response one-form encoding finite recovery mismatch, ledger mismatch, path-dependent update, or non-integrable boundary accounting. Since $d^2\Phi = 0$,

$$d\omega = d\alpha. \quad (34)$$

Equation (33) is the key handoff to H2. H1 does not prove that recovery holonomy generates α. It defines the finite recovery objects from which that claim can be tested.

H1-to-H2 handoff: recovery curvature and projected response

H1 defines the finite recovery objects. H2 tests whether recovery obstruction projects to a screen response curl. The bridge uses three structural steps:

1. **Recovery connection.** The transition maps T_{ij} define a discrete recovery transport. H2 formalizes this as a recovery connection $\Gamma_{\rm rec}$ whose local curvature is
$$F_{\rm rec} = d\Gamma_{\rm rec} + \Gamma_{\rm rec} \wedge \Gamma_{\rm rec}. \quad (35)$$
2. **Projection to screen response.** The residual response component α is a projection of the recovery connection plus non-holonomic contributions:
$$\alpha = P(\Gamma_{\rm rec}) + \alpha_{\rm mem} + \alpha_{\rm leak}, \quad (36)$$
where P projects from recovery connection space to the screen response one-form sector, $\alpha_{\rm mem}$ encodes memory/global holonomy contributions, and $\alpha_{\rm leak}$ encodes leakage contributions not captured by the boundary ledger.
3. **Response curl.** The screen response obstruction is
$$d\omega = d\alpha = P(F_{\rm rec}) + d\alpha_{\rm mem} + d\alpha_{\rm leak}. \quad (37)$$

Not every non-zero recovery holonomy projects to $d\omega \neq 0$. H2 must classify holonomies into four sectors:

1. **Contractible / local curvature**: local mismatches that project to $d\omega$ if $P(F_{\rm rec}) \neq 0$;
2. **Topological / global**: flat but topologically nontrivial — may encode boundary/memory sectors, not local residual;
3. **Gauge / frame artifact**: removed by a change of recovery convention — does not produce physical residual;
4. **Projected-null**: non-zero in recovery space but $P(F_{\rm rec}) = 0$ — does not enter the screen response channel.

Only the first sector (contractible, non-gauge, screen-projected non-zero) contributes to $d\omega$. This sector discipline prevents H2 from overclaiming that any finite recovery obstruction automatically implies a gravitational residual.

H1 supplies the finite recovery objects $(\Gamma_{\rm rec}, F_{\rm rec}, \alpha, P)$. H2 must define the projection P, the connection $\Gamma_{\rm rec}$, and the classification of holonomy sectors. H1 does not prove that recovery holonomy generates $d\omega$.

TABLE II. Holonomy sector classification for the H1-to-H2 bridge.

Sector	**Mathematical status**	**H2/G1 relevance**
Contractible / local curvature	$F_{\rm rec} \neq 0$ on a contractible loop	Projects to $d\omega$ if $P(F_{\rm rec}) \neq 0$ — gravitational residual candidate
Topological / global	Flat but nontrivial monodromy	Boundary/memory sector, does not give local $d\omega$
Gauge / frame artifact	Removable by recovery convention change	No physical residual
Projected-null	$F_{\rm rec} \neq 0$ but $P(F_{\rm rec}) = 0$	Does not enter screen response channel

RT/FLM AS A HIGH-CAPACITY MODEL CLASS

Standard generalized entropy

The RT/HRT and FLM structures relate boundary-region entropy to bulk extremal surfaces and quantum corrections [4–6]:

$$S_{\rm gen}(A) = \frac{{\rm Area}(\gamma_A)}{4G_N} + S_{\rm bulk}(\Sigma_A) + \cdots . \tag{38}$$

H1 does not rederive Eq. (38). Instead, it treats generalized entropy as the cleanest known high-capacity realization of finite screen-capacity accounting.

FDS capacity-ledger reading

The FDS reading uses the bit-capacity ledger. Dividing by $k_B \ln 2$ converts the generalized entropy to a dimensionless capacity:

$$I_{\rm gen}(A) \equiv \frac{S_{\rm gen}(A)}{k_B \ln 2} = I_{\rm area}(A) + I_{\rm bulk}(A) + I_{\rm rec}(A). \tag{39}$$

The terms are interpreted as follows:

- $I_{\rm area}$: minimal screen capacity required to support reconstruction;
- $I_{\rm bulk}$: internal bulk distinction load not represented by the area ledger alone;
- $I_{\rm rec}$: finite-code, finite-recovery, finite-access, or finite-tolerance correction.

This reading is not a replacement for the standard derivations. It is a capacity-accounting reinterpretation of their operational content.

Entanglement wedge

Define the entanglement wedge as a recoverable bulk distinction domain:

$$W_E(A) = \{Q_{\rm bulk} : \epsilon_A(Q) \leq \epsilon_{\rm task}\}. \tag{40}$$

This makes entanglement-wedge reconstruction a special case of finite boundary distinction recovery [7–9].

Islands

Island transitions are ledger-minimization transitions. In FDS language, the winning generalized entropy surface is the boundary ledger that minimizes total recovery cost:

$$\gamma_* = \arg\min_{\gamma} S_{\rm gen}(\gamma). \tag{41}$$

This does not derive islands. It reclassifies them as finite recovery-ledger saddle changes.

Bridge Principle 2 (RT/FLM as capacity-ledger realization)**.** *RT/FLM generalized entropy is the cleanest known asymptotic realization of finite screen-capacity accounting.*

Criterion 3 (Demotion of the RT/FLM bridge)**.** *If generalized entropy cannot be interpreted as a recovery-capacity ledger without losing reconstruction content, this bridge demotes to a heuristic dictionary.*

HOLOGRAPHIC QEC AS BOUNDARY DISTINCTION RECOVERY

Code subspace

Let $\mathcal{H}_{\rm code}$ be a finite bulk code sector. H1 reads it as a recoverable bulk distinction sector:

$$\mathcal{H}_{\rm code} \quad \longleftrightarrow \quad \text{recoverable bulk distinction sector.} \tag{42}$$

The code subspace is not merely a Hilbert subspace. It is the portion of bulk distinction space that the boundary ledger can represent and recover within the declared tolerance.

Boundary redundancy

Boundary redundancy is finite recovery capacity distributed over boundary subledgers. A bulk distinction may be recoverable from multiple overlapping boundary regions, but the redundancy is not free. It consumes capacity, imposes gluing constraints, and can fail under erasure or leakage.

Leakage diagnostic

Define unreconstructable complement information $E_{\text{unreconstructable}}$. The holographic leakage diagnostic is

$$H^{\text{holo}}_{\text{leak}} = \mathrm{I}(Q_{\text{bulk}}; E_{\text{unreconstructable}} \mid R_{\partial A}). \tag{43}$$

This measures bulk distinction information outside the recoverable boundary ledger. It is the holographic analogue of directed leakage in finite recovery models.

Matched-entropy, different-recovery toy models

The central claim of H1 is that boundary entropy alone does not guarantee recoverability. We illustrate this with two minimal finite codes that share the same boundary entropy but differ sharply in recovery error.

Proposition 1 (Matched local entropy, split recovery)**.** *There exist finite boundary ledgers with matched local marginal boundary entropy $H(B_i)$ for each single boundary region B_i but different recovery errors $\epsilon_{B_i}(Q)$ for the same bulk distinction.*

Proof by construction. We exhibit two codes with $H(B_i) = 1$ for every boundary region B_i $(i = 1, 2, 3)$:

Model 1 (Redundancy code — perfect recovery)**.** *Let a single bulk bit $Q \in \{0, 1\}$ be encoded into three boundary bits:*

$$B_1 = B_2 = B_3 = Q. \tag{44}$$

Each boundary bit individually has entropy $H(B_i) = 1$ (matched across codes). Any single boundary bit perfectly recovers Q:

$$\epsilon_{B_i}(Q) = 0. \tag{45}$$

The recovery ledger is trivial: no gluing obstruction, zero leakage.

Model 2 (Parity code — zero single-site recovery)**.** *Let two auxiliary random bits U, V be drawn independently. Encode the bulk bit as*

$$B_1 = U, \quad B_2 = V, \quad B_3 = Q \oplus U \oplus V. \tag{46}$$

Again each boundary bit has $H(B_i) = 1$ (matched local marginal entropy), but any single boundary bit carries zero information about Q:

$$I(Q; B_i) = 0, \qquad \epsilon_{B_i}(Q) = 1/2. \tag{47}$$

Recovering Q requires at least two boundary bits.

These two models establish the core H1 diagnostic:

Matched boundary entropy, split recovery. The redundancy code and parity code have identical single-region boundary entropy $H(B_i) = 1$, yet the recovery error $\epsilon_{B_i}(Q)$ splits from 0 to 1/2. Boundary entropy alone is therefore insufficient to characterize recoverability; the recovery ledger requires redundancy structure, access geometry, and leakage diagnostics.

Extension to quantum tensor-network codes follows the same logic: stabilizer or holographic codes with matched boundary von Neumann entropies can exhibit different reconstruction fidelities depending on the erasure structure and the bulk operator's location relative to the entanglement wedge [10].

Protocol 1 (Matched-entropy, different-recovery test)**.** *Construct finite (classical or quantum) codes with matched boundary entropy $S(A)$ but different redundancy/erasure structure. Measure $\epsilon_A(Q)$ for the same bulk distinction. H1 predicts that recovery can split at matched entropy if the access structure, redundancy, and leakage diagnostics differ.*

THREE HOLOGRAPHIC REGIMES

AdS/CFT

In AdS/CFT, the relevant boundary is an asymptotic timelike boundary. The FDS-H1 status is

$$\text{AdS/CFT} = \begin{array}{l}\text{sharp high-capacity model class in which}\\ \text{recovery deficits and gluing obstructions}\\ \text{are idealized away.}\end{array} \tag{48}$$

This does not reduce AdS/CFT to FDS vocabulary. It identifies where AdS/CFT sits inside the broader finite-screen recovery architecture.

Celestial and soft-gravity holography

Celestial and asymptotically flat programs use null infinity and soft boundary data [13, 14]. H1 reads such structures as asymptotic encodings of finite screen edge distinctions. The finite-diamond phase-space bridge strengthens the motivation for this reading [15].

Finite causal-screen holography

Finite causal-screen holography treats finite causal diamonds, horizon patches, and observer-relative screens as primary. Its boundary is not necessarily an asymptotic

CFT boundary. It is a finite recovery ledger:

$$\text{finite screen} \quad \Rightarrow \quad (\text{capacity}, \text{recovery}, \text{gluing}, \text{response}). \tag{49}$$

Regime table

Criterion 4 (Central H1 reclassification)**.** *AdS/CFT is not rejected. It is reclassified as a sharp high-capacity model class within the broader finite-screen recovery architecture. Subregion-subalgebra duality [24] sharpens the boundary-region-to-bulk correspondence, while finite-N and gauge-invariance cautions [26] are consistent with H1's emphasis that ideal reconstruction does not unconditionally extend to finite screen recovery.*

MINIMAL TOY MODELS AND DIAGNOSTICS

Finite tensor-network recovery model

A minimal H1 model is a finite tensor-network code with boundary regions A_i, local recovery maps $\mathcal{R}_{A_i}$, and recoverable bulk operators Q. One constructs recovery transports T_{ij} between local reconstructions and measures loop mismatch Ω_{ijk}.

This model is not meant to reproduce a full holographic CFT. It is meant to test whether finite recovery nets support meaningful gluing obstruction.

General diagnostic form

The matched-entropy models of the previous subsection illustrate the general diagnostic: hold $S(A)$ fixed while changing boundary redundancy or erasure pattern. In an entropy-only model, recoverability would be determined entirely by $S(A)$. In the H1 ledger model, recovery error can also depend on redundancy structure, leakage, and gluing consistency:

$$\epsilon_A(Q) = F\big(S(A), \Delta_A(Q), H^{\text{holo}}_{\text{leak}}, \Omega_{\text{rec}}\big). \tag{50}$$

Recovery loop diagnostic

For a loop

$$A_1 \to A_2 \to A_3 \to A_1, \tag{51}$$

compute

$$\Omega_{\text{rec}} = T_{31}T_{23}T_{12} - \text{id}. \tag{52}$$

The diagnostic asks whether reconstructed bulk distinctions return identically, up to tolerance, after transport around the loop.

Screen response toy model

Let

$$X = (A, L, \Pi_L) \tag{53}$$

be a simplified screen variable set and let

$$\Phi(X) = S_{\text{scr}}(X)/k_B. \tag{54}$$

Introduce a residual response α and compute

$$\omega = d\Phi + \alpha, \qquad d\omega = d\alpha. \tag{55}$$

The purpose is not to prove G1. It is to show how finite recovery mismatch can be represented as response non-exactness in controlled settings.

Remark 2 (Why toy models matter)**.** *Without a finite recovery model, H1 would risk becoming a broad holographic dictionary. The toy models force the central claims into recoverable objects: capacities, maps, errors, transports, loop mismatch, and response non-exactness.*

SUCCESS CRITERIA AND DEMOTION PATHS

Minimal success criteria

H1 succeeds if it establishes:

1. a finite screen ledger formalism;
2. a definition of screen-relative bulk distinction;
3. a finite recovery-net model;
4. a recovery gluing obstruction;
5. a capacity-ledger reading of RT/FLM;
6. a holographic QEC recovery interpretation;
7. a clear handoff to H2/H3.

H1 does not need to derive M3/4 to succeed.

Demotion paths

H1 claims demote if:

1. bulk recovery cannot be formulated as finite boundary ledger recovery;
2. recovery failure does not track capacity deficit, leakage, or gluing obstruction;
3. finite causal-diamond variables cannot serve as screen variables;

TABLE III. Three holographic regimes in the FDS-H1 classification.

Regime	**Boundary object**	**FDS-H1 status**	**Main limitation**
AdS/CFT	Asymptotic timelike boundary	Sharp high-capacity model class; recovery deficits idealized away	Not generic for finite observers or cosmological screens
Celestial / soft gravity	Null infinity / soft data	Asymptotic encoding of finite screen edge distinctions	Requires finite-screen interpretation to become observer-local
Finite causal-screen holography	Finite causal diamond or causal screen	Primary finite-boundary object	Requires H2/H3 to connect to $d\omega$ and residual cosmology

4. RT/FLM cannot be read as capacity accounting without losing reconstruction meaning;
5. recovery nets do not support meaningful holonomy or gluing diagnostics;
6. no controlled model connects recovery mismatch to response non-exactness.

Branch independence

H1 success does not prove G1. G1 success does not prove H1. M3/4 success does not prove H1. M3/4 failure does not falsify H1. This branch-local discipline prevents the H-series from becoming an unconstrained unification narrative.

APPENDIX: INTERNAL FDS CROSSWALK (NOT USED AS PREMISE)

This appendix is not part of the H1 argument. It records internal terminology alignment only. H1 depends only on finite screen ledgers, recovery maps, capacity constraints, and gluing diagnostics.

H1 shares finite-boundary obstruction logic with other FDS modules, but uses distinct objects:

- Q0 studies boundary-access holonomy H_∂ and the quantum reversible quotient. H1's recovery holonomy Ω_{rec} is a holographic gluing object, not the quantum-access holonomy of Q0.
- Q2 studies protected logical distinctions and QEC maintenance. H1 imports standard holographic QEC recovery logic without requiring Q2's specific code-maintenance framework.
- G1 studies finite causal-screen entropy response $\omega = \eta_i(X)dX^i$. H1 supplies candidate screen variables and capacity ledgers but does not prove G1's response equations; H2/H3 handle the onward bridge.

H1 success does not prove G1. G1 success does not prove H1. M3/4 success does not prove H1. M3/4 failure does not falsify H1.

H-SERIES ROADMAP

H1

H1 defines finite causal-screen holography. Its central objects are

$$C_\Sigma, \quad S_\Sigma, \quad \mathcal{R}_A, \quad \epsilon_A, \quad \Omega_{\text{rec}}. \tag{56}$$

Its goal is to define a finite boundary recovery architecture.

H2

H2 will address recovery holonomy and screen-response curl. Its central object is the recovery connection Γ_{rec} and its curvature F_{rec}. H2 must define the projection P from recovery curvature to the screen response one-form sector and test whether

$$P(F_{\text{rec}}) \neq 0 \Rightarrow d\omega \neq 0 \tag{57}$$

under the holonomy sector discipline: only contractible, non-gauge, screen-projected non-zero curvature contributes. Global, topological, gauge, and projected-null holonomies are excluded. H1 supplies the finite recovery objects; H2 supplies the connection, the projection, and the sector classification.

H3

H3 will address whether finite-screen recovery can constrain the M3/4 branch under a restricted optical-screen recovery class:

$$\begin{aligned} d\omega \;\Rightarrow\; \kappa &= \frac{2\chi_S + \chi_\Omega}{\chi_A + 2\chi_S + \chi_\Omega}, \\ \mu = 1, \qquad \Sigma - 1 &= -\frac{3}{4}(3 - s)R_{bH}(a). \end{aligned} \tag{58}$$

H3 requires the stated optical-screen assumptions (isotropic compliance, Ward-suppressed Ricci leakage, passive kernel, no free $A(a,k)$) and is not assumed by H1. Its failure does not falsify H1.

TABLE IV. Division of labor in the three-paper H-series.

Paper	**Core object**	**Role**	**Main theorem target**
H1	Finite screen ledger, recovery net, gluing obstruction	Define finite causal-screen holography	Recovery gluing consistency in finite models (exact + tolerant)
H2	Recovery connection $\Gamma_{\rm rec}$, curvature $F_{\rm rec}$	Test whether recovery obstruction projects to $d\omega$	$P(F_{\rm rec}) \neq 0 \Rightarrow d\omega \neq 0$ under holonomy sector discipline
H3	Optical screen residual branch	Test whether finite-screen obstruction can constrain M3/4	$d\omega \Rightarrow \kappa{=}3/4, \mu{=}1, \Sigma$ channel under restricted assumptions

CONCLUSION

FDS-H1 introduces finite causal-screen holography as a finite boundary recovery theory. It shifts holography from ideal asymptotic duality to finite screen ledgers: bulk distinctions are physically accessible relative to a screen only when recoverable through finite boundary capacity, redundancy, and recovery structure.

The core architecture is

$$\boxed{\begin{aligned}&\text{finite screen capacity} \to \text{bulk distinction recovery}\\ &\to \text{recovery net} \to \text{gluing obstruction} \to \text{future } d\omega \text{ bridge.}\end{aligned}} \tag{59}$$

H1 does not derive G1 or M3/4. It builds the objects needed for H2 and H3.

Recovery-Net Formalism

This appendix records the minimal algebraic structure of a finite recovery net.

A recovery net consists of:

1. a finite cover $\mathcal{U} = \{A_i\}$ of a screen Σ;
2. boundary data objects $\mathcal{B}_{A_i}$;
3. recovery maps $\mathcal{R}_{A_i}$;
4. recovered sectors $\mathcal{Q}(A_i)$;
5. overlap consistency tolerances $\epsilon_{\rm glue}$;
6. transition maps $T_{ij} : \mathcal{Q}(A_i) \to \mathcal{Q}(A_j)$;
7. loop holonomies Ω_γ.

A strict recovery sheaf would require exact restriction and gluing. H1 uses a finite tolerant version because physical recovery is error-bounded and task-relative. The exact limit is recovered when $\epsilon_{\rm task} = \epsilon_{\rm glue} = 0$.

For a loop γ, the finite recovery error is

$$\|\Omega_\gamma\|_{D,Q} = D(Q, (T_\gamma Q))\,, \tag{60}$$

where T_γ is the product of transition maps around the loop. A sector is ϵ-flat if

$$\|\Omega_\gamma\|_{D,Q} \leq \epsilon \tag{61}$$

for all admissible loops and all recoverable distinctions Q in the sector.

Tensor-Network Toy Model

A minimal simulation can be built from a finite graph tensor network with boundary legs partitioned into regions A_i. Bulk logical operators are inserted at internal vertices. Local recovery maps are constructed by optimizing reconstruction fidelity from each boundary region.

The simulation protocol is:

1. Fix a bulk operator Q.
2. Choose three overlapping boundary regions A_1, A_2, A_3.
3. Compute local reconstructions $\mathcal{R}_{A_i}(Q)$.
4. Construct transition maps T_{ij} by best matching local recovered representatives.
5. Compute $\Omega_{123} = T_{31}T_{23}T_{12} - \mathrm{id}$.
6. Compare $\|\Omega_{123}\|$ against reconstruction error and boundary entropy.

The null expectation is that entropy alone controls recovery. The H1 expectation is that two networks with matched boundary entropy can differ in loop mismatch and recovery quality because their finite recovery ledgers differ.

RT/FLM Capacity-Ledger Glossary

Notation Summary

AI ASSISTANCE DISCLOSURE

This manuscript draft may have used AI-assisted editing and LaTeX organization under the author's direction. The conceptual claims, mathematical framing, scope boundaries, and scientific responsibility remain with the author.

TABLE V. Holographic objects and FDS-H1 interpretation.

Standard object	**FDS-H1 interpretation**	**Status**
RT/HRT surface	Minimal recovery screen	Bridge interpretation
Area term	Minimal screen capacity ledger	Bridge interpretation
Bulk entropy	Residual internal distinction load	Bridge interpretation
Entanglement wedge	Recoverable bulk distinction domain	Operational reading
Island	Ledger-minimizing saddle transition	Bridge interpretation
Code subspace	Recoverable bulk distinction sector	Operational reading
Boundary erasure	Capacity loss / recovery deficit	Operational diagnostic
Complement leakage	Unrecoverable distinction export	Operational diagnostic

TABLE VI. Notation used in H1.

Symbol	**Meaning**
Σ	finite screen or boundary
$\mathcal{L}_\Sigma$	finite screen ledger
$\mathcal{B}_\Sigma$	accessible boundary data algebra or record set
C_Σ	finite screen capacity
S_Σ	screen entropy ledger
$\mathcal{A}_\Sigma$	access / encoding map from bulk to boundary
$\mathcal{R}_\Sigma$	recovery map from boundary to bulk distinction
Q_{bulk}	bulk distinction
$\epsilon_A(Q)$	recovery error for distinction Q from region A
$R^{\mathcal{T}}_{\min}(Q, \epsilon, \tau)$	minimal task-relative recovery demand
$\Delta^{\mathcal{T}}_{\Sigma}(Q)$	finite screen capacity deficit (task-relative)
$\mathcal{Q}(A)$	sector recoverable from boundary region A
T_{ij}	recovery transition map from region A_i to A_j
Ω_{rec}	recovery holonomy / gluing obstruction
ω	G1 screen response one-form
α	residual response component
$d\omega$	screen-response obstruction
$H^{\text{holo}}_{\text{leak}}$	holographic leakage diagnostic

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