Mathematical Form of Physical Law as Invariant-Form Compression: Invariant, Equivariant, and Covariant Law Forms in Finite Distinction Systems

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Physical laws are remarkably mathematical: they appear as equations, variational principles, symmetries, conservation laws, operators, groups, manifolds, spectra, tensors, and constants. FDS-X5 interprets this fact inside Finite Distinction Systems. It does not claim that mathematics is an external substance governing reality, nor that FDS derives all mathematical structures used in physics. Instead, it proposes that the mathematical form of physical law is the compressed form of invariant, equivariant, or covariant relations that survive finite capacity, perturbation, coordinate change, coarse-graining, and boundary-maintenance constraints.

In an active finite distinction system, raw microstate histories are too large to represent. What persists across observers, scales, coordinates, gauges, and perturbations must factor through a compressed sector. In the strict invariant case,

$$q : X \to Q, \qquad q(P_i x) = q(x),$$

while in the equivariant or covariant case,

$$q(P_i x) = \rho_i q(x), \qquad F(q) = 0 \Rightarrow F(\rho_i q) = 0.$$

A physical law is therefore an operationally stable relation on compressed law-form sectors, rather than a full enumeration of microstates. The "mathematical effectiveness" of physical law is reframed: mathematics is effective because the portable part of physics is the part compressible into invariant-form structures. X5 separates a core FDS claim from higher-risk philosophical extensions. The core claim is that finite boundary-maintaining systems must compress stable physical regularities through invariant, equivariant, or covariant representations. The higher-risk bridge is that Wigner's puzzle is dissolved because persistent physical law and mathematical structure are two perspectives on invariant-form compression.

Reader contract. X5 is not a proof that the universe is literally "made of mathematics." It does not derive all of mathematics from physics, derive the Standard Model or general relativity, solve open problems in pure mathematics, or claim that every mathematically consistent structure is physically realized. It does not claim that every useful effective model is an exact law. It does not turn constants such as e, i, or π into numerological "cosmic signatures." X5 is a claim about the operational form in which finite observers, agents, instruments, and boundary-maintaining systems can discover, store, transmit, and use physical laws. It is not a claim that the universe itself consults a compressed formula before evolving.

Claim-status summary

Table I separates formal FDS claims, physical bridges, and higher-risk interpretive extensions.

Keywords

Finite Distinction Systems; Distinction Theory; invariant-form compression; physical law; Wigner puzzle; covariance; symmetry; conservation laws; Noether theorem; rate-distortion; algorithmic information; renormalization; operators; Hilbert space; constants; computability; Physical AI.

INTRODUCTION

Wigner's puzzle, reframed

Wigner's puzzle asks why mathematics is so effective in the natural sciences [9]. Complex numbers became central to quantum mechanics; Riemannian geometry became the language of general relativity; group theory and representation theory became indispensable in particle physics. The puzzle is usually phrased as a mystery about the relation between human formal constructions and external reality.

X5 reframes the question. The relevant issue is not why all mathematics describes physics. Most mathematical structures are not directly realized in physical law. The sharper question is why the stable, portable, predictive

TABLE I. Central X5 claims, status, and failure or demotion conditions.

Claim	Status	What would weaken or falsify it
Finite systems cannot internally represent all microstate detail.	Formal FDS core	A finite system internally represents unbounded environmental distinctions with no compression, externalization, task relaxation, or hidden capacity.
Stable law-like regularities require invariant-form compression.	FDS structural claim	Portable physical regularities persist with no invariant, equivariant/covariant form, symmetry, quotient, stable statistic, or compressed relation.
Mathematical equations are compressed representations of invariant-form relations.	Interpretive bridge	Exact physical laws require no compression, no form stability, no stable relational structure, and no finite representational economy.
Symmetries reduce rule-maintenance cost.	Physical / information bridge	Asymmetric local rule tables are physically cheaper and more stable than invariant or equivariant compressed rules under the same task requirements.
Wigner's puzzle is reframed by invariant-form compression.	Philosophical bridge	Mathematics remains effective for law-like physics where no invariant/equivariant/covariant compression, symmetry, operator structure, quotient, or stable relation exists.
e, i, and related constants are model-class signatures.	Optional bridge	Their appearance is asserted as universal without model-class assumptions or is shown to be purely conventional in the relevant context.
Open mathematical problems may have physical analogues.	Speculative appendix	Analogues are mistaken for derivations, predictions, or proofs.

part of physics has mathematical form. In FDS terms, the answer is:

> Physical laws are mathematical because law-like regularities are invariant-form compressions.

Raw histories are too large for finite systems to represent. What can be maintained, communicated, predicted, and reused must be compressed. What remains stable across perturbations must preserve form. Mathematical law is the symbolic form of this invariant, equivariant, or covariant compression.

Physical law as finite description

A physical law is not a database of all microstates. It is a finite rule that compresses many possible histories into a relation that remains valid across an equivalence class of states, frames, scales, gauges, or perturbations. In a finite distinction system, a raw history

$$x_{0:T} \in X^{T+1} \tag{1}$$

is useful only if it can be replaced, for the task at hand, by a smaller quotient, sufficient statistic, invariant, covariant object, or operator representation

$$x_{0:T} \mapsto q(x_{0:T}) \in Q. \tag{2}$$

A law is the compressed rule on Q and its transformation law, not the raw enumeration of X.

X5 in the X-series

X2 analyzed the algebraic lower bound needed for CP/T-oriented weak identity transformation. X3 interpreted the four known interactions as a functional distinction-operation closure. X4 interpreted Pauli exclusion as collision-free fermionic address protection. X5 explains why these claims naturally appear as equations, algebras, symmetries, nilpotencies, operators, and constants: physical theory records invariant-form compression structures of finite matter and finite observation.

FDS BACKGROUND

Active finite distinction systems

An active finite distinction system is represented by

$$\mathcal{S} = (X, E, B, M, Y, A, U, \pi, \ell, \Phi, \mathcal{P}, \tau), \tag{3}$$

where X is internal state, E environment, B boundary, M memory/model state, Y observation channel, A action space, U update rule, π finite projection, ℓ boundary-maintenance loss, Φ resource budget, $\mathcal{P}$ perturbation/pruning family, and τ update timescale [1]. The FDS core separates formal claims, physical bridges, and quarantined applications; X5 uses the formal capacity and invariant-persistence layers rather than claiming a direct metaphysical derivation of physics.

Capacity deficit and rate-distortion demand

For a finite memory or model space M, internal representational capacity is

$$C_S = \log_2 |M|. \tag{4}$$

For an admissible task family Ψ, the rate-distortion demand over update window τ at tolerance ε is $R^{(\tau)}_{\min}(\varepsilon;\Psi)$. The FDS capacity deficit is

$$\Delta_\varepsilon(\tau) = R^{(\tau)}_{\min}(\varepsilon;\Psi) - C_S. \tag{5}$$

If $\Delta_\varepsilon(\tau) > 0$, the system must approximate, externalize, relax the task, improve compression, or fail [1]. X5 asks what form the best compression takes when the regularity must remain portable across perturbations and coordinate choices.

Invariant-supported persistence

Let $R_A : X \to \{0,1\}$ be a structural identity predicate. An identity is invariant-supported under perturbation family $\mathcal{P}$ when there exist a quotient $q : X \to T$ and a predicate $\bar{R}_A : T \to \{0,1\}$ such that

$$R_A = \bar{R}_A \circ q, \qquad q \circ P_i = q \quad \forall P_i \in \mathcal{P}. \tag{6}$$

Then $R_A \circ P_i = R_A$ for every admissible perturbation [1]. X5 generalizes this into a law-form claim: law-like physical regularities factor through compressed sectors that may be strictly invariant, equivariant, or covariant. Strict invariants are the simplest case; tensor equations, gauge laws, and quantum representations often preserve form by transforming covariantly rather than by remaining numerically unchanged.

DEFINITIONS

Definition 1 (Physical regularity). *A physical regularity is a relation R over states, observations, or histories,*

$$R \subseteq X \times X', \tag{7}$$

or a predictive map $f : X \to Y$ that improves prediction, control, compression, or boundary-maintenance loss relative to a registered task.

Definition 2 (Law-like regularity). *A physical regularity is law-like relative to perturbation family $\mathcal{P}$ when it remains valid under admissible transformations, or when it factors through a compressed sector $q : X \to Q$:*

$$R(x, x') = \bar{R}(q(x), q(x')). \tag{8}$$

A law-like regularity is not merely a repeated pattern; it is a portable pattern under a specified equivalence, covariance, or perturbation class.

Definition 3 (Invariant-form compression). *An invariant-form compression is a map $q : X \to Q$ together with a law relation F and a registered transformation family $\mathcal{P}$ satisfying three properties:*

$$\begin{aligned} &\textit{compression:} && L_{\mathrm{rep}}(q;\mathcal{D}) \ll L_{\mathrm{raw}}(\mathcal{D}),\\ &\textit{task sufficiency:} && \mathbb{E}[\ell(Z,\hat{Z}(q(X)))] \le \varepsilon,\\ &\textit{form stability:} && q(P_i x) = q(x) \ \textit{or}\ q(P_i x) = \rho_i q(x). \end{aligned} \tag{9}$$

Here $\mathcal{D}$ is the registered task-relevant data, history ensemble, rule table, model state, or experiment class. $L_{\mathrm{raw}}(\mathcal{D})$ is the description length of the raw representation under the chosen coding scheme. $L_{\mathrm{rep}}(q;\mathcal{D})$ is the description length of the compressed representation plus decoder, law form, or rule needed to use it.

Definition 4 (Strict invariant, equivariant, and covariant cases). *In the strict invariant case,*

$$q(P_i x) = q(x). \tag{10}$$

In the equivariant or covariant case, admissible transformations act on the compressed sector through a representation ρ_i:

$$q(P_i x) = \rho_i q(x), \tag{11}$$

while the law relation is preserved in form:

$$F(q) = 0 \quad \Rightarrow \quad F(\rho_i q) = 0. \tag{12}$$

This distinction is essential for physics: tensor equations, gauge fields, quantum states, and relativistic laws often transform covariantly rather than remaining componentwise invariant.

Definition 5 (Mathematical law form). *A mathematical law form is a finite symbolic representation of an invariant, equivariant, or covariant relation. Common forms include*

$$F(q) = 0, \qquad \dot{q} = \mathcal{L}q, \qquad G \curvearrowright Q, \qquad I(q) = \text{constant}. \tag{13}$$

Equations, tensor relations, symmetry laws, conservation laws, variational principles, operator algebras, spectra, and topological invariants are law forms when they compress stable physical regularities.

Definition 6 (Law compression ratio). *For a registered dataset, history ensemble, rule table, or model state $\mathcal{D}$ encoded directly with length $L_{\mathrm{raw}}(\mathcal{D})$, a law representation with length L_{law}, and initial/boundary data length L_{data}, define the normal-form law compression ratio*

$$\mathcal{C}_{\mathrm{law}} = \frac{L_{\mathrm{raw}}(\mathcal{D})}{L_{\mathrm{law}} + L_{\mathrm{data}}}. \tag{14}$$

This is an audit measure, not a universal constant. High $\mathcal{C}_{\mathrm{law}}$ means strong mathematical compression relative to the specified coding scheme and task class.

MAIN PROPOSITION: LAW AS INVARIANT-FORM COMPRESSION

Proposition 1 (Physical law as invariant-form compression)**.** *Consider an active finite distinction system maintaining a predictive or boundary-relevant regularity over perturbation family $\mathcal{P}$. If raw state detail exceeds the system's finite representational capacity, then any portable law-like representation must factor through a compressed task-sufficient sector*

$$X \xrightarrow{q} Q \xrightarrow{\bar{f}} \hat{Z}, \tag{15}$$

where q is strict-invariant, equivariant, covariant, or stable to the registered tolerance under the admissible transformations relevant to the law. The mathematical form of the law is the finite description of q, $\bar{f}$, the representation ρ_i when present, and the preserved form relation $F(q) = 0 \Rightarrow F(\rho_i q) = 0$.

Proof. Finite capacity bounds internally maintained detail by C_S. If the raw task-relevant state or history requires more than C_S bits, full enumeration is unavailable. Boundary maintenance still requires a statistic that preserves the task-relevant information to tolerance ε. Thus the system must use a compressed representation $q(X)$. If the regularity is to remain portable across the admissible transformation family, the relevant content must either be strictly invariant, transform equivariantly or covariantly under a known representation, or remain stable to the registered tolerance. Hence prediction or control factors through Q and its transformation law. The symbolic description of that quotient, representation, update rule, invariant relation, or group action is what is called the mathematical form of the law. □

Remark 1 (Core versus bridge)**.** *The proposition is an FDS structural statement about finite representation and invariant-form quotients. It concerns operational law form for finite systems, not an ontological claim that the universe consults a compressed formula. It does not imply that every exact law has already been discovered, that all mathematics is physically realized, or that the physical world is reducible to a human formalism.*

WHY EQUATIONS?

Equations as invariant-form constraints

An equation records a constraint that remains true across an equivalence class. For example,

$$F(q) = 0 \tag{16}$$

says that the compressed variable q lies in a law-defined subspace. Conservation laws, gauge constraints, mass-shell conditions, and commutation relations all have this form: they reduce a large set of possible states to a smaller law-defined manifold.

The equation is not merely notation. It is a compression device. Instead of storing every allowed case, one stores the relation that recognizes all allowed cases. In covariant theories, the components of q may change with coordinates or gauge, but the equation form is preserved; the compression is in the stable relation, not necessarily in invariant component values.

Dynamical equations as compressed update rules

A dynamical equation

$$\dot{q} = F(q) \tag{17}$$

compresses many trajectories into one reusable update rule. In a linear generator model,

$$\dot{q} = \mathcal{L}q, \qquad q(t) = e^{\mathcal{L}t}q(0). \tag{18}$$

The exponential form appears because repeated homogeneous updates are compressed by a semigroup generator. This is a model-class bridge, not a claim that every physical process is exactly Markovian or linear. When the generator description is valid, $e^{\mathcal{L}t}$ is the invariant-form compression of repeated time-translation updates.

Differential equations as continuum compression

Differential equations are often continuum or coarse-grained limits of local update regularities. They are effective when many discrete or microscopic updates can be summarized by smooth fields and local derivatives. Thus differential equations are not mystically effective; they are local invariant-form compression rules for domains whose finite update dynamics admits a smooth approximation.

WHY SYMMETRY?

Symmetry and covariance as rule compression

Let a group G act on X. A strict G-invariant quotient satisfies

$$q(x) = q(gx), \qquad g \in G. \tag{19}$$

An equivariant or covariant representation satisfies

$$q(gx) = \rho(g)q(x), \tag{20}$$

with the law relation preserved in form. Instead of writing separate rules for every point in the orbit Gx, a symmetric law writes one rule on the orbit space, representation space, or covariant tensor relation. This reduces rule-table size, parameter count, and coordinate dependence.

Cost of asymmetry

A fully asymmetric law must specify extra local rule data: preferred positions, labels, directions, frames, exceptions, or contexts. In finite systems, those specifications consume memory, communication, and update resources. Symmetry is therefore not merely aesthetic. It is a compression strategy: one rule applies across many cases. Recent work in symmetry-informed equation discovery shows that equivariance constraints can compress the equation-search space and improve robustness and simplicity of discovered governing equations [23].

Noether bridge

Noether's theorem states that continuous symmetries of an action correspond to conserved quantities [10, 11]. X5 does not derive Noether's theorem. It interprets why Noether-type structures are privileged: conservation laws are invariant-form compressions of dynamical redundancy. They let a finite system track a smaller number of stable quantities instead of the full trajectory. This interpretation is aligned with recent sparse-invariant methods for discovering interpretable conservation laws [24] and with geometric frameworks for Hamiltonian systems that preserve symmetry under Lie group transformations [25].

WHY CONSTANTS?

Constants as operational scales

A physical constant is not merely a decorative number in an equation. In law-form compression it functions as a stable operational scale: a causal conversion limit, a phase-action scale, a coupling strength, an entropy-energy conversion factor, or a boundary invariant.

$$\theta(q) = \theta_0. \tag{21}$$

The constant compresses many measurements into one reusable conversion scale or coupling value. If the parameter drifted arbitrarily, the system would need to store a table $\theta(x, t, \text{context})$ rather than a law. Dimensional constants depend on unit conventions and conversion scales. Their invariant content lies in dimensionless combinations, operational ratios, transformation roles, or stable coupling structure, not in their numerical value in a chosen unit system. The invariant role of each constant lies in the operational constraint it imposes: c sets the causal signal-propagation limit, $\hbar$ sets the quantum phase-action scale, G sets the stress-energy geometry coupling, and k_B sets the entropy-energy conversion factor.

e as semigroup signature

In generator models,

$$\frac{dq}{dt} = Lq \Rightarrow q(t) = e^{Lt}q(0). \tag{22}$$

The number e should not be over-interpreted as a mystical constant. The physically meaningful object is the exponential functional form: the eigenfunction structure of homogeneous generator evolution. The numerical base is tied to units and conventions; the compression principle is semigroup update.

i as coherent phase bookkeeping

In quantum theory,

$$U(t) = e^{-iHt/\hbar}. \tag{23}$$

The imaginary unit is the algebraic bookkeeping device for phase-preserving coherent dynamics, interference, and unitary evolution. X5 does not claim to derive Hilbert-space quantum mechanics from FDS alone. It says that once a domain supports coherent superposition and phase-sensitive composition, complex operators are an invariant-form compression language for that domain.

Anti-numerology firewall for π and other constants

Some constants appear because of geometry, coordinate conventions, unit choices, or model-specific expansions. The appearance of π in circular geometry does not by itself make π a cosmic signature. The deeper invariant may be boundary topology, isoperimetric geometry, or rotational symmetry; the numerical coefficient depends on convention and dimension. X5 treats constants as law-compression parameters only under explicit model-class assumptions.

WHY OPERATORS, HILBERT SPACES, AND SPECTRA?

Operators as compressed transition rules

An operator $\mathcal{O} : X \to X$ encodes a reusable transformation. Instead of enumerating many input-output pairs, the operator applies one rule across a domain. In this sense, operators are transition-rule compression.

Hilbert spaces as superposition-compatible compression

Hilbert spaces compress amplitude relations, superpositions, measurement bases, and transformations in quantum theory. X5 does not derive the Hilbert-space formalism. It gives an operational reading: when a physical domain supports coherent superposition and compositional amplitudes, Hilbert-space operators are the mathematical compression of stable amplitude transformations.

Spectra as compressed observable structure

A spectrum replaces a large family of possible responses with eigenvalues and eigenvectors:

$$\mathcal{O}\psi_n = \lambda_n \psi_n. \tag{24}$$

Spectral structure is a powerful law form because it compresses dynamics, measurement outcomes, relaxation modes, stability, and resonance into indexed invariant data.

WHY PURE MATHEMATICS OFTEN PRECEDES PHYSICS

Mathematics explores possible compression forms

Pure mathematics studies possible structures without requiring their physical instantiation. From the X5 perspective,

Mathematics is the search space of possible invariant-form compressions.

Physics selects those forms that are compatible with measurement, stability, finite resources, perturbation survival, and empirical boundary conditions.

Renormalization-group language gives a standard physical example of this selection principle. Under repeated coarse-graining, most microscopic details flow away, while a smaller set of relevant variables, symmetries, scaling relations, and fixed-point structures remains predictive. In X5 terms, an infrared fixed point is a stable invariant-form compression: a relation whose form survives coarse-graining and therefore becomes law-like for finite observers. X5 does not reduce all physical law to RG fixed points; RG is a model-class example of invariant-form compression under scale coarse-graining.

Finding an invariant-form compression may require an expensive search over hypotheses, symmetries, coordinates, latent variables, and equivalence classes. Once found, however, the law form can be applied, stored, communicated, and verified at much lower marginal cost. This asymmetry explains why law-like compressions have high finite-system value: they amortize discovery cost across many future predictions.

Selection principle

A mathematical structure becomes physically relevant only if it compresses a real regularity, survives the relevant perturbations, can be instantiated or approximated under finite resources, and improves prediction or boundary maintenance. Mathematics is larger than physics because it explores possible compressions; physics realizes only the compressions that can be paid for and maintained.

Wigner's puzzle reframed

Wigner's puzzle is not answered by saying "all mathematics is physics." It is answered by selection:

$$\text{physical law} \subset \text{maintainable invariant-form compressions}. \tag{25}$$

Mathematical structures often precede physics because mathematics explores invariant forms before nature, experiment, and physical theory identify which forms are instantiated.

OPTIONAL BRIDGE: OPERATIONAL COMPUTABILITY

This section is not required for the main invariant-form compression thesis. It is an optional bridge concerning the form in which finite-resolution physical predictions can be operationally used by finite systems.

Conjecture 1 (Operational computability). *Operational physical laws governing finite-resolution measurements are expected to be computably approximable to registered task tolerance.*

The reason is FDS-like: a finite system cannot use an infinite oracle table as a law. A law that is predictive for finite observers must be finitely specifiable or finitely approximable within the required tolerance and update window. This does not prove that all mathematical objects appearing in physics are computable in every idealization. It states a physical bridge: operational law use requires finite procedures.

Criterion 1 (Computability demotion). *The X5 computability bridge is weakened if a reproducible finite-resolution physical law has predictive content that provably requires noncomputable functions and cannot be replaced by any computable approximation within observational tolerance.*

SPECULATIVE APPENDIX: MATHEMATICAL PROBLEMS AS POSSIBLE COMPRESSION ANALOGUES

Firewall. No mathematical problem is claimed to be solved, predicted, or physically derived in this appendix. The purpose is only to illustrate how some mathematical structures may acquire physical analogues when they appear as spectra, partition functions, complexity gaps, or symmetry classes. This appendix is not part of the X5 core claim.

Riemann hypothesis

Spectral interpretations of zeta zeros have long motivated connections between number theory and physics. X5 allows a weak statement: if a zeta-like object appears as the spectral determinant of a physical operator, then properties of its zeros may acquire physical meaning. This is not a derivation of the Riemann hypothesis.

P versus NP

In FDS language, generation and verification need not have the same cost. A structure may be easy to check but hard to construct. This resembles, but does not prove, a generation-verification asymmetry familiar from computational complexity. X5 does not prove complexity-class separations.

Finite simple groups

The classification of finite simple groups catalogs possible symmetry building blocks. X5 treats such mathematics as part of the structure space from which physics may select maintainable symmetries. It does not predict that every group is physically realized.

RELATION TO P-SERIES AND X-SERIES

Table II places X5 in the local FDS corpus.

NUMERICAL AND CONCEPTUAL DEMONSTRATIONS

The figures are deterministic normal-form demonstrations generated by `code/generate_results.py`. They illustrate definitions and compression intuitions rather than fitting empirical data.

FALSIFICATION AND DEMOTION CONDITIONS

X5 is weakened if stable physical laws are found that cannot be represented by any invariant, equivariant, covariant, compressed relation, stable quotient, symmetry, operator, or sufficient statistic. It is weakened if finite systems maintain law-like prediction without reducing raw microstate complexity, or if physical regularities persist across perturbations while lacking any form-stable structure.

The constants bridge is demoted whenever a claimed constant is shown to be a coordinate convention, unit convention, or model-specific artifact. The computability bridge is demoted if reproducible finite-resolution predictions provably require noncomputable law content. The Wigner interpretation can fail without destroying the FDS core: the core claim is finite invariant-form compression; the metaphysical interpretation is a higher-level bridge.

RELATION TO EXISTING THEORY

X5 uses existing theories as implementation bridges rather than replacing them. Wigner's essay supplies the target problem [9]. Noether's theorem supplies a canonical example in which symmetry compresses dynamics into conserved quantities [10, 11]. Rate-distortion theory and information theory supply the compression backbone [12, 13]. Algorithmic information theory and minimum-description-length methods supply law-code and model-selection analogues [14–16]. Renormalization group theory supplies a scale-dependent model of coarse-grained invariant persistence [17]. Symmetry and group theory supply orbit compression [18]. Dynamical systems and semigroups supply generator and flow forms [19]. Quantum theory supplies operator, Hilbert-space, and spectral law forms [20]. The FDS Core supplies the finite-capacity and invariant-persistence basis [1]. Covariance and tensorial law forms connect this compression account to the standard geometric language of physical theory [22].

DESIGN IMPLICATION FOR PHYSICAL AI

This section is not part of the physics proof. It extracts the FDS design rule implied by X5: a physical AI agent should learn not more correlations, but better invariant, equivariant, and covariant compressions.

TABLE II. Relation of X5 to neighboring FDS papers.

Paper	Core role	X5 relation
FDS Core	finite capacity, capacity deficit, invariant quotients	formal basis for invariant-form compression [1]
P3	environmental side records and Markovianization	laws ignore inaccessible bath detail by quotienting [2]
P4	anti-recurrence / lost preimages	laws compress away erased detail rather than recover it [3]
P6	finite update windows and resource ledgers	laws reduce update throughput and verification burden [4]
P7	invariant side-ledgers / topological protection	law form as protected quotient structure [5]
X2	CKM phase-counting lower bound	example of algebraic physical constraint [6]
X3	four-interaction operation closure	interactions become mathematical through invariant roles [7]
X4	Pauli nilpotency as address protection	algebraic law as occupancy-protection compression [8]
X5	mathematical law form	invariant-form compression of physical regularities

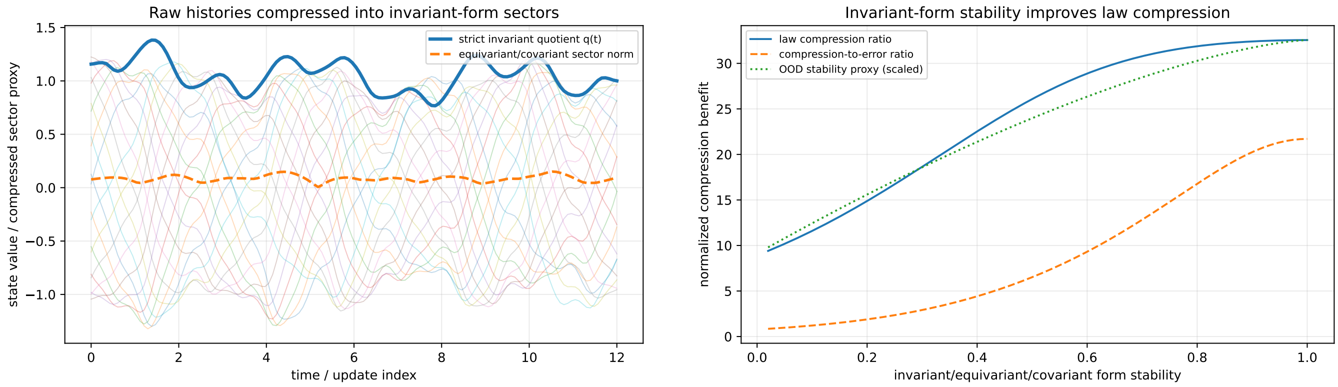

FIG. 1. Left: raw histories with different micro-details compress into strict-invariant and equivariant/covariant sectors. Right: as form-stability strength increases, a normal-form law compression ratio and compression-to-error ratio improve. The OOD stability proxy (scaled) illustrates that invariant-form compressions maintain predictive power under distribution shift. Out-of-distribution stability, latent constraint preservation, and compression-to-error ratio serve as audit metrics for compression quality. These are conceptual demonstrations, not data fits.

Physics	AI analogue
Physical law	invariant world-model rule
Symmetry / covariance	equivariance / data augmentation
Conservation law	latent constraint
Equation	compact predictive program
Constant	stable parameter
Operator	reusable transition rule
Invariant quotient	task-preserving latent representation

An agent that stores raw histories without discovering invariants or equivariances pays excessive memory, update, and verification cost. Failure modes include overfitting, missing symmetry, brittle out-of-distribution behavior, high update cost, and lack of conserved latent structure.

The generalization gap in many learned systems can be read as a failure of invariant-form compression. A correlational map may fit observed histories while failing under distribution shift because it has not identified the quotient, symmetry, conservation law, or covariant structure that remains stable outside the training distribution. This failure mode is also recognized in physics-informed machine learning, where incorporating physical priors (symmetries, conservation laws, operator structures) improves generalizability and physical plausibility [26]. Useful audit metrics include out-of-distribution stability, latent constraint preservation, and compression-to-error ratio.

FDS-X5 therefore suggests a benchmark: report invariant compression quality, not only prediction accuracy.

CONCLUSION

The mathematical form of physical law is not an external miracle. In FDS terms, it is the finite symbolic form of invariant-form compression. A physical law is not a complete enumeration of the world's microstates. It is a compact representation of relations that remain stable under perturbation, coordinate change, coarse-graining, and boundary-maintenance demands. Equations, symmetries, conservation laws, tensor relations, operators, spectra, and constants are the forms taken by such compressions.

X5 does not derive all mathematics or replace physics with metaphysics. It identifies why the stable part of physics is naturally mathematical: only invariantly, equivariantly, or covariantly compressible structure can

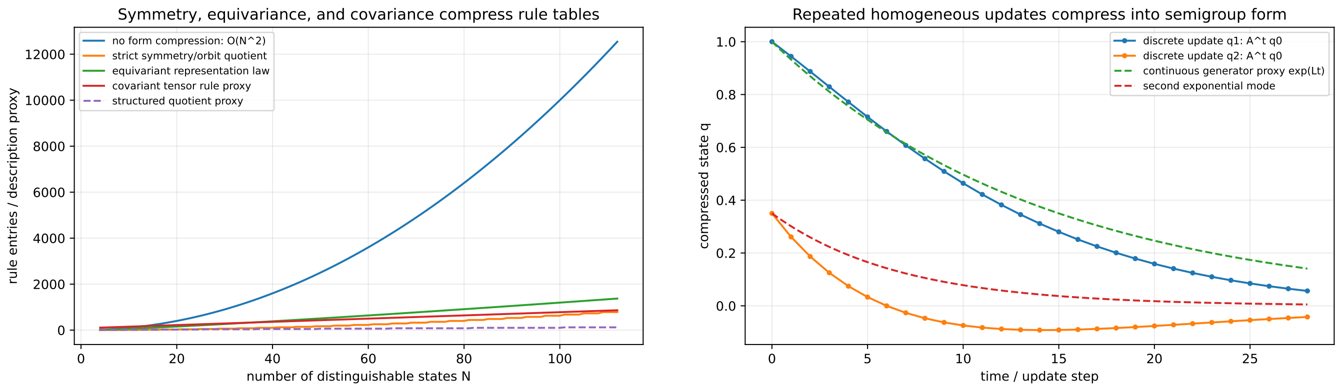

FIG. 2. Left: symmetry and equivariance compress rule tables by reducing cases to orbits or representation laws. For a fully asymmetric rule table, specification scales with the number of cases; a symmetry or covariance group reduces the maintained rule table to orbit-level data plus the group action. The plotted curves are normal-form scaling illustrations, not empirical parameter counts. Right: repeated homogeneous updates can be compressed as $A^t q_0$ or, in a generator limit, $e^{Lt} q_0$. The exponential is a semigroup-compression signature, not a numerological claim about e.

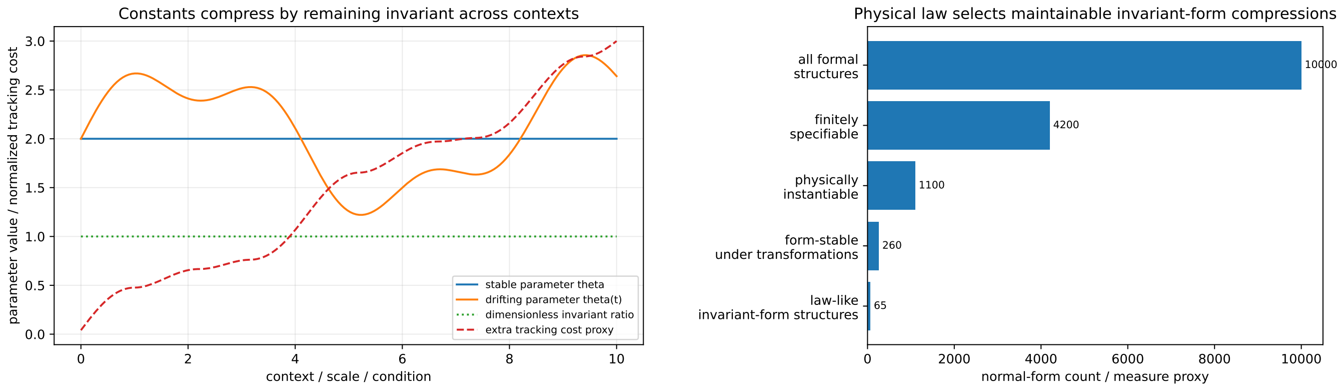

FIG. 3. Left: a stable parameter compresses many contexts into one constant; drifting parameters require extra tracking cost. Right: Wigner's puzzle reframed as a selection filter with FDS bottlenecks: physics realizes only mathematical structures that are finitely specifiable (finite symbolic update), physically instantiable (Φ, resources), stable at finite resolution (C_S, memory), stable under perturbation ($\mathcal{P}$, invariance), and real-time usable (τ, P6 throughput).

be retained, communicated, predicted, and maintained by finite systems. The unreasonable effectiveness of mathematics becomes less mysterious when physical law is understood as the portable mathematical residue of finite distinguishability.

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