Minimal Nonzero CP/T Orientation and Full Weak-Sector Consistency
CKM-PMNS Orientation Capacity, Texture Independence, and the Three-Family Threshold in Finite Distinction Systems

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The hard theorem in FDS-X2 is the Dirac orientation-capacity theorem: for an $N \times N$ charged-current mixing matrix, the number of independent rephasing-invariant Dirac CP/T orientation phases is

$$C_{\text{orient}}(N) = \frac{(N-1)(N-2)}{2}.$$

Therefore $N = 3$ is the unique minimal dimension carrying exactly one primitive Dirac orientation. The theorem applies canonically to the CKM sector and treats the PMNS sector as a full weak-sector consistency extension: if the Dirac-type PMNS charged-current interface also demands one primitive CP/T orientation, then the same capacity threshold gives $N_\ell = 3$. Thus, under the finite-minimality bridge,

$$D_q = D_\ell = 1 \quad \Longrightarrow \quad N_q = N_\ell = 3.$$

The paper separates orientation capacity from mixing texture: CKM and PMNS may have radically different angle patterns while sharing the same minimal $N = 3$ Dirac-orientation capacity. Majorana phases, if present, are treated separately as identity-locking or lepton-number boundary data, not as ordinary oscillation Dirac orientation. The result is a conditional minimality and consistency theorem, not a derivation of fermion masses, Yukawa matrices, CKM or PMNS angles, baryogenesis, anomaly cancellation, the neutrino mass mechanism, or the Standard Model gauge group.

Reader Contract. This paper does not claim that FDS derives three fermion generations unconditionally. It identifies a conditional minimality bridge: one primitive Dirac CP/T orientation demand in a charged-current identity-update sector requires the minimal flavor dimension $N = 3$. The CKM sector supplies the canonical application. The PMNS sector is a conditional full-sector extension, not an independent proof of leptonic CP violation, PMNS texture, neutrino masses, or the neutrino mass mechanism.

VERSION GENEALOGY

FDS-X2 v2.0 is the integrated canonical version combining the minimal nonzero CP/T orientation theorem with the full weak-sector PMNS extension.

INTRODUCTION

The flavor-dimensionality question

The observed weak charged-current flavor architecture contains three active quark families and three active lepton families. In the quark sector, the CKM matrix is close to diagonal. In the lepton sector, the PMNS matrix has large mixing angles. These very different textures coexist with the same active family count. FDS-X2 asks a narrow question:

> What is the minimal flavor dimension required for a charged-current identity-update sector to carry exactly one primitive rephasing-invariant CP/T orientation?

The answer is algebraic: for a Dirac-type $N \times N$ mixing matrix, the number of independent physical CP/T orientation phases is $(N-1)(N-2)/2$. The first nonzero case is $N = 3$, and it carries exactly one orientation phase.

What is hard and what is conditional

The hard component of the paper is standard rephasing algebra. The conditional component is the FDS bridge that interprets a weak charged-current mixing matrix as a finite identity-update sector whose task demand includes one primitive CP/T orientation channel. The strongest canonical result is therefore:

$$D_q = 1 \quad \Longrightarrow \quad N_q = 3 \tag{1}$$

for the CKM sector, under finite orientation minimality. The PMNS extension asks whether the lepton charged-current sector is consistent with the same capacity threshold. If its Dirac-type identity-update demand is also one, then

$$D_q = D_\ell = 1 \quad \Longrightarrow \quad N_q = N_\ell = 3. \tag{2}$$

This is a full weak-sector consistency theorem, not a proof that CKM and PMNS have similar textures.

Claim-status summary

WEAK IDENTITY-UPDATE SECTORS

CKM charged-current identity update

The quark charged current contains

$$J^{\mu}_{\mathrm{cc,q}} = \bar{u}_i \gamma^{\mu}(1-\gamma^5) V_{ij} d_j, \tag{3}$$

where $V = V_{\mathrm{CKM}}$ is the CKM matrix. In FDS language, this is an identity-update interface between down-type and up-type weak flavor records:

$$\mathcal{T}_q : |d_j\rangle \mapsto \sum_i V_{ij} |u_i\rangle. \tag{4}$$

A physical complex phase that survives rephasing supplies orientation data for this identity transformation.

PMNS charged-current identity update

The lepton charged current contains

$$J^{\mu}_{\mathrm{cc},\ell} = \bar{\ell}_{\alpha} \gamma^{\mu}(1-\gamma^5) U_{\alpha k} \nu_k, \tag{5}$$

where $U = U_{\mathrm{PMNS}}$ is the PMNS matrix. The same Dirac rephasing-counting algebra applies to the Dirac-type oscillation phase. In FDS terms, the PMNS interface connects neutrino propagation eigenstates to charged-lepton flavor records under weak interaction.

Remark 1 (Shared capacity, different dynamics)**.** *The CKM and PMNS sectors share the same Dirac orientation-capacity formula. They do not share the same mass hierarchy, angle texture, or sector dynamics. FDS-X2 constrains dimension and orientation capacity, not the numerical values of mixing matrix entries.*

DIRAC ORIENTATION CAPACITY

Rephasing-invariant orientation

Definition 1 (Dirac CP/T orientation)**.** *A Dirac-type charged-current mixing sector carries physical CP/T orientation when it admits a nonzero rephasing-invariant imaginary quartet,*

$$J_{ij;kl} = \mathrm{Im}\left(U_{ik} U_{jl} U^*_{il} U^*_{jk}\right), \tag{6}$$

with nonzero relevant mixing angles and nondegenerate mass structure. Removable phases are conventions. Nonzero invariants of Eq. (6) *are physical orientation data.*

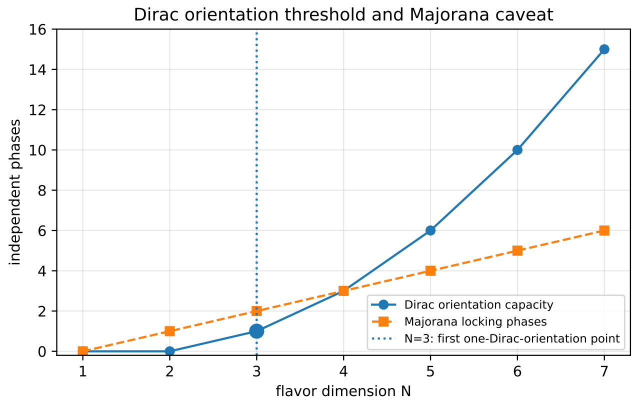


FIG. 1. Dirac orientation capacity and Majorana caveat. The Dirac capacity first becomes nonzero at $N = 3$ and equals one only at $N = 3$. Majorana phases, if present, are counted separately as locking data.

Parameter count

An $N \times N$ unitary matrix has N^2 real parameters. These may be decomposed into

$$N_{\mathrm{angle}} = \frac{N(N-1)}{2} \tag{7}$$

angles and

$$N_{\mathrm{phase,raw}} = \frac{N(N+1)}{2} \tag{8}$$

raw phases. For Dirac fields, $2N-1$ phases are removable by independent field rephasings. Therefore the number of physical Dirac CP/T orientation phases is

$$C_{\mathrm{orient}}(N) = \frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}. \tag{9}$$

Theorem 1 (Dirac orientation-capacity theorem)**.** *For an $N \times N$ Dirac-type charged-current mixing matrix,*

$$C_{\mathrm{orient}}(N) = \frac{(N-1)(N-2)}{2}. \tag{10}$$

Thus $N = 1, 2$ carry no primitive Dirac CP/T orientation, $N = 3$ carries exactly one, and $N > 3$ carries multiple independent orientation phases.

Proof. The formula follows from the rephasing count above. The cases are immediate: $C_{\mathrm{orient}}(1) = 0$, $C_{\mathrm{orient}}(2) = 0$, $C_{\mathrm{orient}}(3) = 1$, and $C_{\mathrm{orient}}(N) > 1$ for every integer $N > 3$. ☐

TABLE I. Central FDS-X2 v2.0 claims, status, and demotion conditions.

Claim	**Status**	**Demotion or failure condition**
$C_{\text{orient}}(N) = ((N-1)(N-2))/2$	Standard algebra	Standard Dirac rephasing count fails.
$N = 3$ uniquely gives one Dirac phase	Algebraic theorem	$N = 2$ has one, or $N = 3$ lacks one.
CKM identity update demands $D_q = 1$	FDS bridge	Quark weak update needs none or more than one.
$D_q = 1 \Rightarrow N_q = 3$	Canonical result	Required fourth active quark family is nonredundant.
PMNS shares Dirac capacity algebra	Extension	Lepton interface is not PMNS-like.
$D_\ell = 1 \Rightarrow N_\ell = 3$	Conditional extension	Leptonic Dirac orientation is absent or not required.
$D_q = D_\ell = 1 \Rightarrow N_q = N_\ell = 3$	Full-sector consistency	Active families beyond three are required.
Majorana phases are separate locking data	Scope firewall	Majorana phases are treated as oscillation Dirac phases.
CKM/PMNS texture equality	Not claimed	The paper is read as predicting equal textures.

MINIMAL NONZERO ORIENTATION THEOREM

Definition 2 (Primitive orientation demand). *Let $r \in \mathbb{N}$ denote the number of primitive independent Dirac CP/T orientation channels demanded by an audited weak identity-update task. A sector with dimension N satisfies the demand if*

$$C_{\text{orient}}(N) \geq r. \tag{11}$$

Theorem 2 (Minimal dimension by orientation demand). *The minimal Dirac-type flavor dimension satisfying a primitive orientation demand $r \geq 1$ is*

$$N_{\min}(r) = \left\lceil \frac{3 + \sqrt{1 + 8r}}{2} \right\rceil. \tag{12}$$

In particular, $N_{\min}(1) = 3$.

Proof. Solve $((N-1)(N-2))/2 \geq r$, i.e.

$$N^2 - 3N + 2 - 2r \geq 0. \tag{13}$$

The positive-root threshold is $(3 + \sqrt{1+8r})/2$. Taking the ceiling gives the minimal integer dimension. For $r = 1$, this gives $N = 3$. ☐

Corollary 1 (Unique minimal one-orientation architecture). *A Dirac-type charged-current sector carrying exactly one primitive CP/T orientation has minimal dimension $N = 3$. Dimensions $N < 3$ lack the channel; dimensions $N > 3$ carry surplus independent orientation capacity.*

CANONICAL X2 RESULT: CKM SECTOR

Assumption 1 (CKM primitive orientation demand). *The audited CKM weak identity-update sector demands one primitive Dirac CP/T orientation:*

$$D_q = 1. \tag{14}$$

This does not assert that CKM CP violation alone explains baryogenesis or fixes CKM angles. It asserts only that the quark charged-current identity-update interface requires one irreducible orientation channel.

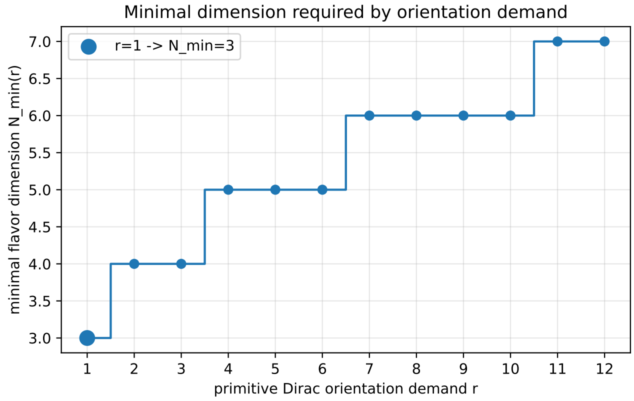


FIG. 2. Minimal flavor dimension required by primitive Dirac orientation demand r. The case $r = 1$ yields $N_{\min} = 3$.

Criterion 1 (Finite orientation minimality). *A finite weak identity-update sector does not maintain independent primitive orientation capacity beyond the task demand unless the extra capacity supplies specified nonredundant physical work.*

Theorem 3 (Canonical CKM X2 result). *Under the CKM primitive-orientation demand $D_q = 1$ and finite orientation minimality, the minimal CKM-participating quark-family dimension is*

$$N_q = 3. \tag{15}$$

Proof. By Eq. (12), one primitive orientation demand has minimal dimension $N_{\min}(1) = 3$. Dimensions one and two have zero capacity; dimensions greater than three contain surplus independent orientation phases. Finite minimality selects the minimal satisfying dimension, $N_q = 3$. ☐

FULL WEAK-SECTOR CONSISTENCY EXTENSION

Conditional PMNS orientation demand

For a Dirac-type PMNS oscillation phase, the same orientation-capacity theorem applies. The lepton-sector extension assumes

$$D_\ell = 1, \tag{16}$$

namely that the PMNS charged-current interface demands one primitive Dirac CP/T orientation. This is conditional because the leptonic Dirac phase remains an experimental target and because the neutrino mass mechanism may introduce additional structure.

Theorem 4 (PMNS conditional extension)**.** *If the Dirac-type PMNS charged-current sector demands one primitive orientation,* $D_\ell = 1$*, and finite orientation minimality applies, then the minimal PMNS-participating active lepton dimension is*

$$N_\ell = 3. \tag{17}$$

Proof. The PMNS Dirac-type phase obeys the same rephasing-counting algebra as Eq. (9). With $D_\ell = 1$, Eq. (12) gives $N_\ell = 3$. ☐

Full-sector consistency theorem

Theorem 5 (Full weak-sector orientation consistency)**.** *If the quark and lepton charged-current identity-update sectors each demand one primitive Dirac CP/T orientation,*

$$D_q = D_\ell = 1, \tag{18}$$

and both are subject to finite orientation minimality, then

$$N_q = N_\ell = 3. \tag{19}$$

Proof. Apply the canonical CKM result to $D_q = 1$ and the PMNS conditional extension to $D_\ell = 1$. ☐

Consistency, not PMNS proof

The full-sector theorem is not a proof that leptonic CP violation is already experimentally established. If future oscillation data force the PMNS Dirac phase to 0 or π, then the PMNS sector still has the $N = 3$ capacity to carry one Dirac orientation, but the assumption $D_\ell = 1$ is demoted. The quark-sector theorem remains unaffected.

TEXTURE INDEPENDENCE

The CKM and PMNS matrices have very different absolute-value textures. This is not a problem for X2 because the theorem concerns phase capacity, not matrix entries. Texture is controlled by Yukawa matrices, mass hierarchies, the neutrino mass mechanism, and sector-specific dynamics. Orientation capacity is controlled by flavor dimension and rephasing algebra.

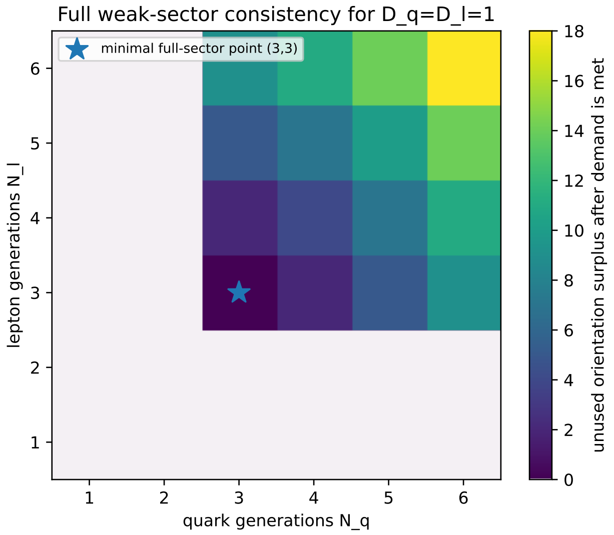
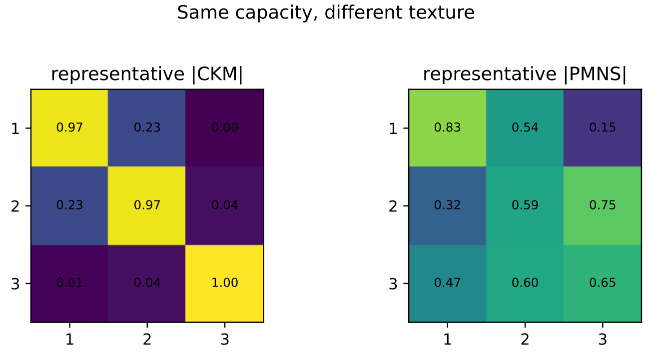


FIG. 3. Full weak-sector consistency for $D_q = D_\ell = 1$. Gray cells fail at least one sectoral orientation demand. Colored cells satisfy both but may carry surplus orientation capacity. The star marks the minimal full-sector point $(N_q, N_\ell) = (3, 3)$.



	1	2	3
1	0.97	0.23	0.00
2	0.23	0.97	0.04
3	0.01	0.04	1.00

	1	2	3
1	0.83	0.54	0.15
2	0.32	0.59	0.75
3	0.47	0.60	0.65

FIG. 4. Representative absolute-value textures. CKM is close to diagonal while PMNS is broadly mixed. X2 constrains the minimum dimension for one Dirac orientation channel; it does not predict equal CKM and PMNS angle patterns. Values are illustrative and are not a fit.

Criterion 2 (Texture independence)**.** *Two weak charged-current sectors may share the same orientation capacity,*

$$C^{q}_{\mathrm{orient}}(3) = C^{\ell}_{\mathrm{orient}}(3) = 1, \tag{20}$$

while having very different mixing textures:

$$|V^{\mathrm{CKM}}_{ij}| \not\sim |U^{\mathrm{PMNS}}_{ij}|. \tag{21}$$

FDS-X2 predicts equality of minimal orientation capacity, not equality of mixing angles.

MAJORANA CAVEAT

If neutrinos are Dirac particles, the PMNS phase count follows the same Dirac rephasing algebra as CKM. If neutrinos are Majorana particles, individual neutrino field rephasings are restricted. An N-flavor Majorana PMNS matrix contains

$$C_{\mathrm{orient}}(N) = \frac{(N-1)(N-2)}{2} \tag{22}$$

Dirac-type phases and

$$C_{\mathrm{Maj}}(N) = N - 1 \tag{23}$$

additional Majorana phases. For $N = 3$, this gives one Dirac phase and two Majorana phases.

X2 treats these separately. The Dirac phase controls ordinary oscillation CP/T orientation. Majorana phases, if physical, are additional lepton-number or identity-locking boundary data relevant to lepton-number-violating processes such as neutrinoless double beta decay. They are not counted as ordinary oscillation Dirac orientation channels.

Remark 2 (Majorana firewall)**.** *The PMNS extension fails if it treats Majorana phases as the same object as the Dirac oscillation phase. Majorana phases may be important for lepton-number boundary structure, but they do not replace the Dirac orientation-capacity theorem.*

RELATION TO FAMILY PACKAGING

Observed active weak fermions are organized into repeated chiral family packages. X2 does not derive anomaly cancellation and does not claim that anomaly cancellation fixes the number of families. It uses family packaging only as context for why quark and lepton generation counts are naturally compared:

$$N_{\mathrm{gen}} = N_q = N_\ell \tag{24}$$

in the observed active weak sector. The FDS contribution is different: it asks why the repeated weak family package realizes the minimal nonzero orientation capacity, namely $N = 3$.

NORMAL-FORM COST ILLUSTRATION

A normal form illustrates the finite-minimality bridge without replacing it. Let

$$K_{\mathrm{mix}}(N) = (N-1)^2 \tag{25}$$

be a sectoral mixing-overhead proxy, and let

$$B(N) = \min\{C_{\mathrm{orient}}(N), 1\} \tag{26}$$

be a benefit that saturates after the first required primitive orientation. A full-sector illustrative cost is

$$\mathcal{F}(N_q, N_\ell) = \alpha[K_{\mathrm{mix}}(N_q) + K_{\mathrm{mix}}(N_\ell)] + \beta(N_q + N_\ell) \tag{27}$$

$$+ \kappa(N_q - N_\ell)^2 - \lambda[B(N_q) + B(N_\ell)], \tag{28}$$

with positive coefficients. The packaging term penalizes quark-lepton active-family mismatch. Eq. (28) is not a fundamental flavor potential; it is a normal form for surplus-orientation cost once one primitive orientation per sector is enough.

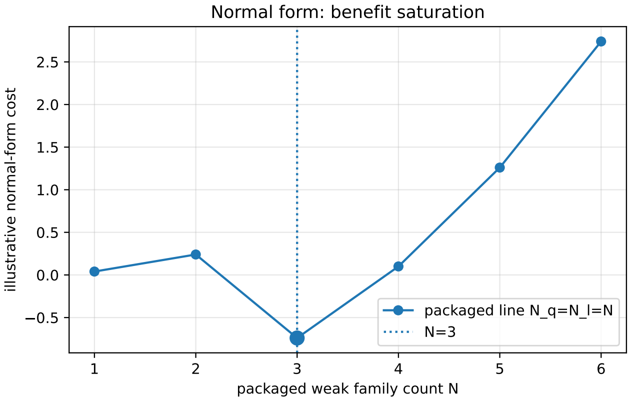


FIG. 5. Illustrative full-sector normal-form cost along the packaged line $N_q = N_\ell = N$. Orientation benefit saturates after one primitive channel per sector, while overhead grows with extra capacity. This figure is not a fit and not a fundamental theory of flavor.

BOUNDARIES OF THE CLAIM

Fourth generations

A required fourth active sequential chiral generation would demote the exact-three bridge unless the additional family can be shown to be redundant, decoupled, vector-like, hidden-sector, or outside the audited charged-current orientation task. If a fourth active family supplies nonredundant physical work or lowers the relevant finite-system cost, the minimality bridge must be revised.

Vector-like matter

X2 concerns active sequential chiral generations participating in CKM-like and PMNS-like charged-current architectures. It does not exclude vector-like matter or additional hidden-sector fermions. Such sectors require separate FDS mappings.

Sterile neutrinos

A sterile neutrino that does not participate in the standard charged current is not automatically counted in N_ℓ for PMNS orientation capacity. If an additional active neutrino is confirmed and participates in an enlarged unitary PMNS-like charged current, then N_ℓ and the orientation capacity must be redefined.

Baryogenesis, strong CP, and other CP sectors

Neither CKM nor PMNS orientation capacity is claimed to be sufficient for the observed baryon asymmetry. Strong CP, extended scalar CP phases, leptogenesis phases, and beyond-Standard-Model CP sources are distinct sectors. X2 supplies a dimensional and orientation-capacity constraint, not a complete CP theory.

Masses and mixing angles

The theorem does not compute masses, Yukawa matrices, CKM angles, PMNS angles, the neutrino mass ordering, or the neutrino mass mechanism. These belong to sector-specific dynamics beyond orientation capacity.

FALSIFICATION AND DEMOTION CONDITIONS

The algebraic core fails only if standard rephasing parameter counting for Dirac-type charged-current mixing is wrong. The canonical CKM bridge is demoted if quark weak identity transformation requires no primitive CP/T orientation or requires more than one. The PMNS extension is demoted if leptonic Dirac orientation is not required, is not realized, or is not PMNS-like in the relevant charged-current sense. The full-sector exactly-three bridge is demoted if required active generations beyond three are established and shown to supply nonredundant physical work or lower the relevant finite-system cost. None of these failures would by itself falsify the FDS formal core.

NUMERICAL AND ALGEBRAIC DEMONSTRATIONS

The accompanying script `code/generate_results.py` generates the orientation-capacity table, minimal-dimension table, representative CKM/PMNS texture plot, full-sector consistency grid, and normal-form cost figure. The algebraic capacity has no fitted parameters. The CKM and PMNS matrices used in Fig. 4 are representative absolute-value textures, not fits. The normal-form cost uses illustrative coefficients and is not a flavor potential.

CONCLUSION

FDS-X2 v2.0 integrates the minimal nonzero CP/T orientation theorem with the full weak-sector PMNS extension. The hard algebraic core is

$$C_{\text{orient}}(N) = \frac{(N-1)(N-2)}{2}, \qquad N_{\min}(r) = \left\lceil \frac{3+\sqrt{1+8r}}{2} \right\rceil . \tag{29}$$

For one primitive Dirac orientation demand, the unique minimal dimension is three. The canonical CKM bridge gives $D_q = 1 \Rightarrow N_q = 3$. The conditional PMNS extension gives $D_\ell = 1 \Rightarrow N_\ell = 3$. Together,

$$D_q = D_\ell = 1 \quad \Longrightarrow \quad N_q = N_\ell = 3. \tag{30}$$

This is a minimality and consistency result, not a derivation of Standard Model flavor dynamics. It explains why CKM and PMNS may share the same minimal orientation-capacity threshold while differing radically in texture.

CODE AVAILABILITY

The script `code/generate_results.py` regenerates all figures, CSV tables, and the model summary JSON included with this release.

AI ASSISTANCE DISCLOSURE

AI-assisted tools were used for language polishing, structural feedback, LaTeX drafting support, and code-debugging assistance. The author reviewed and edited all content and remains responsible for all claims, references, simulations, and conclusions. No AI system is listed as an author.

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