Horizon-Maintenance Dark Energy and Horizon Ledger Occupancy: Pre-Euclid Registration, the Minimal $w(a)$ Relation, and Reconstruction Protocols in Finite Distinction Systems

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FDS-X1 is a conditional physical bridge claim connecting finite observer boundaries to the dark-energy scale. The original pre-Euclid note registered the claim before major Euclid dark-energy data products: if a cosmological horizon is treated as a finite distinguishability boundary, the relevant dark-energy scale should be horizon-like, of order $M_{\rm Pl}^2 H_0^2$, rather than a Planck-volume vacuum density. The present version integrates both roles: pre-registration and technical model, sharpening the claim into a horizon-ledger occupancy model. Using the standard horizon entropy and temperature as bridge inputs, a spherical horizon of radius R_h has ledger density

$$\rho_h = \frac{T_h S_h}{V_h} = 3M_{\rm Pl,red}^2 R_h^{-2}$$

in natural units. The dark-energy-like horizon-maintenance component is written

$$\rho_{HM}(a) = 3M_{\rm Pl,red}^2 \mu_h(a) R_h^{-2}(a),$$

where $\mu_h(a)$ is a bounded horizon-ledger occupancy or response factor. If this component obeys an effective continuity equation, its physical equation of state satisfies

$$w_{HM}(a) = -1 - \frac{1}{3}\frac{d\ln\mu_h}{d\ln a} + \frac{2}{3}\frac{d\ln R_h}{d\ln a}.$$

The strict non-phantom version is equivalent to the ledger-growth condition $d\ln\mu_h/d\ln a \le 2d\ln R_h/d\ln a$. The paper gives an event-horizon specialization, a reconstruction protocol for $\mu_h(a)$, pre-registered outcome and demotion conditions, and normal-form numerical code. It does not derive general relativity, quantum gravity, the Bekenstein-Hawking coefficient, perturbations, or a likelihood-ready replacement for ΛCDM.

Reader Contract. This paper is a conditional physical bridge in the FDS/DT program. It is not a completed cosmological theory. It uses horizon entropy, horizon temperature, finite causal access, and boundary-ledger accounting as bridge inputs. It does not claim to derive the exact numerical value of Λ from the distinction primitive alone, nor does it claim that any deviation from ΛCDM supports FDS-X1. Its purpose is to integrate the original pre-Euclid prediction registry with the horizon-ledger occupancy model and to make the resulting claim auditable.

VERSION NOTE

Version 1.0 was the pre-Euclid priority registry: it stated the horizon-maintenance dark-energy claim, the non-phantom physical expectation, and demotion conditions before major Euclid dark-energy results. Version 1.1 was the technical sharpening: it introduced the horizon-ledger density, occupancy factor, and minimal $w(a)$ relation. Version 1.2 is the integrated canonical version:

$$\boxed{\text{pre-Euclid registry} + \text{horizon-ledger occupancy model.}} \tag{1}$$

The original pre-Euclid note remains the priority timestamp. This version is the main technical-preregistration statement.

CLAIM STATUS AND FAILURE PROPAGATION

INTRODUCTION

Why integrate the two X1 versions?

The pre-Euclid X1 note had a strategic role: register the claim and its possible failure before decisive survey data. Its strength was not a detailed cosmological likelihood model, but a timestamped outcome contract. The technical model turns the horizon-scale claim into a minimal mathematical model. It replaces the broad statement $\rho \sim M_{\rm Pl}^2 H^2$ with a boundary formula,

$$\rho_{\rm HM}(a) = 3M_{\rm Pl,red}^2 \mu_{\rm h}(a) R_{\rm h}^{-2}(a). \tag{2}$$

TABLE I. FDS-X1 v1.2 claim-status hierarchy.

Claim	Status	Demotion or failure condition
Dark-energy scale is horizon-like rather than Planck-volume-like	Pre-registered physical bridge	If future data and theory require a bulk vacuum density explanation unrelated to horizon/access accounting.
Horizon ledger density $T_h S_h/V_h = 3M_{\mathrm{Pl,red}}^2 R_h^{-2}$	Conditional bridge calculation	If the selected boundary does not admit the horizon entropy/temperature inputs.
$\rho_{HM} = 3M_{\mathrm{Pl,red}}^2 \mu_h R_h^{-2}$	Minimal technical model	If μ_h cannot be bounded, reconstructed, or tied to a physical horizon/access boundary.
Minimal $w(a)$ relation	Conditional theorem	If the horizon-maintenance component does not satisfy an effective continuity equation.
Strict non-phantom physical sector	Pre-registered prediction	If robust model-independent reconstruction requires a true physical phantom sector.
Strict ΛCDM result	Demotion condition	If high-precision data favor constant ρ_{DE} with no evidence for horizon response, the dynamic X1 version is demoted to a static boundary interpretation.
Domain failure does not kill FDS core	Failure propagation rule	X1 failure demotes this physical bridge only; it does not falsify the FDS formal core.

The present paper combines these roles. X1 becomes a constrained horizon-ledger reconstruction program, not merely a scaling analogy.

Current observational context

DESI reported in March 2025 that its first three years of data strengthen hints that dark energy may be evolving, while its guide emphasizes the interpretation of DR2 BAO measurements and evolving-dark-energy parameter constraints [15, 16]. Euclid's first worldwide data release is planned for October 2026 and is expected to include core-science results relevant to dark energy [17, 18]. X1 is written to be auditable against this kind of upcoming evidence rather than retrofitted after the fact.

What is not claimed

X1 does not claim that FDS derives general relativity, quantum gravity, the Bekenstein-Hawking coefficient, the exact value of Λ, or the perturbation sector of a complete dark-energy model. It also does not claim that DESI has confirmed X1 or that every departure from ΛCDM is evidence for X1. The claim is narrower: if dark energy is a horizon-maintenance component, then its density, equation of state, and reconstruction should obey the ledger relations below.

RELATED WORK AND NOVELTY BOUNDARY

Cosmic acceleration was discovered through Type Ia supernovae [4, 5] and is well described in the standard model by a cosmological constant with $w = -1$ [6]. The cosmological constant problem arises because naive quantum-field vacuum estimates are enormously larger than the observed dark-energy density. X1 does not solve that microphysical cancellation problem. It asks whether the observed scale is naturally boundary-like rather than bulk-like.

Horizon entropy and temperature are standard bridge inputs from black-hole and de Sitter thermodynamics [7–9]. Jacobson's thermodynamic derivation of Einstein's equation motivates the idea that horizon thermodynamics and gravitational dynamics are deeply related [10]. Holographic dark energy and UV/IR arguments already contain the scaling $\rho \sim M_{\mathrm{Pl}}^2 L^{-2}$ [11–14]. X1 does not claim novelty for that scaling itself. Its novelty is the finite-distinguishability interpretation, the ledger occupancy factor, the non-phantom ledger condition, and the pre-registered demotion protocol.

FINITE-DISTINGUISHABILITY SETUP

Definition 1 (Cosmological distinguishability boundary)**.** *A cosmological distinguishability boundary for observer patch O is a horizon-like surface beyond which physical differences cannot be operationally registered, preserved, and used by O within the relevant causal, thermodynamic, and finite-record constraints.*

Assumption 1 (Horizon thermodynamic bridge)**.** *For the target boundary, the entropy and temperature inputs are*

$$S_{\mathrm{h}} = \frac{k_{\mathrm{B}} c^3 A_{\mathrm{h}}}{4G\hbar}, \qquad T_{\mathrm{h}} = \frac{\hbar c}{2\pi k_{\mathrm{B}} R_{\mathrm{h}}}. \tag{3}$$

In natural units, $S_{\mathrm{h}} = A_{\mathrm{h}}/(4G)$ and $T_{\mathrm{h}} = (2\pi R_{\mathrm{h}})^{-1}$. These are standard physical inputs, not FDS-derived theorems.

TABLE II. Novelty boundary relative to holographic dark energy and horizon thermodynamics.

Aspect	**Prior art**	**FDS-X1 addition**
$\rho \sim M_{\rm Pl}^2 L^{-2}$	Holographic and UV/IR dark-energy literature	Not claimed as novel; reinterpreted as finite distinguishability boundary density.
Horizon entropy and temperature	Black-hole and de Sitter thermodynamics	Used as physical bridge inputs, not derived from FDS.
Cutoff selection	Model-dependent in HDE	Constrained by causal access, recordability, and boundary-maintenance meaning.
Equation of state	Depends on model	Written as $w = -1 - (1/3)d\ln\mu_h/d\ln a + (2/3)d\ln R_h/d\ln a$.
Failure conditions	Often implicit	Pre-registered demotion and falsification table.

Definition 2 (Horizon-ledger occupancy)**.** *The function $\mu_{\rm h}(a)$ is the dimensionless occupancy or response factor of the selected horizon ledger. It measures how much of the selected boundary ledger appears as the dark-energy-like horizon-maintenance sector. The strongest saturated case is $\mu_{\rm h} = 1$ for the chosen boundary. Weaker versions require only that $\mu_{\rm h}(a)$ be physically bounded, reconstructible, and horizon-tied.*

HORIZON LEDGER DENSITY

Theorem 1 (Horizon ledger density)**.** *For a spherical horizon of radius $R_{\rm h}$, entropy and temperature given by Eq. (3), and volume $V_{\rm h} = 4\pi R_{\rm h}^3/3$, the ledger density associated with $T_{\rm h}S_{\rm h}$ is*

$$\rho_h = \frac{T_{\rm h}S_{\rm h}}{V_{\rm h}} = \frac{3c^4}{8\pi G R_{\rm h}^2}. \tag{4}$$

In natural units,

$$\rho_h = 3M_{\rm Pl,red}^2 R_{\rm h}^{-2}, \qquad M_{\rm Pl,red}^2 \equiv \frac{1}{8\pi G}. \tag{5}$$

Proof. For $A_{\rm h} = 4\pi R_{\rm h}^2$,

$$S_{\rm h} = \frac{\pi k_{\rm B} c^3 R_{\rm h}^2}{G\hbar}. \tag{6}$$

Thus

$$T_{\rm h}S_{\rm h} = \frac{\hbar c}{2\pi k_{\rm B} R_{\rm h}} \frac{\pi k_{\rm B} c^3 R_{\rm h}^2}{G\hbar} = \frac{c^4 R_{\rm h}}{2G}. \tag{7}$$

Dividing by $4\pi R_{\rm h}^3/3$ gives Eq. (4). Eq. (5) follows from the definition of $M_{\rm Pl,red}$. □

Corollary 1 (Area suppression)**.** *Up to convention-dependent Planck-density factors,*

$$\frac{\rho_h}{\rho_{\rm Pl}} \propto \left(\frac{\ell_{\rm P}}{R_{\rm h}}\right)^2. \tag{8}$$

For a cosmic horizon $R_{\rm h}/\ell_{\rm P} \sim 10^{61}$, this gives the familiar 10^{-122} scale as a boundary-area suppression, not a Planck-volume sum.

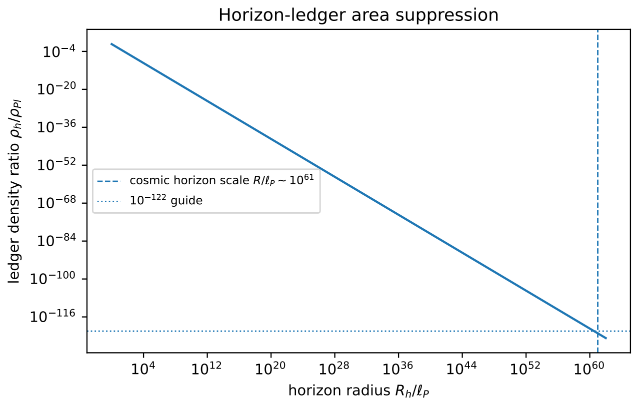

FIG. 1. Horizon-ledger area suppression. The density scale decreases as R_h^{-2} rather than as a Planck-volume bulk sum. This is a scaling illustration, not a derivation of the exact observed value of Λ.

The horizon-maintenance component is defined by

$$\boxed{\rho_{\rm HM}(a) = 3M_{\rm Pl,red}^2 \mu_{\rm h}(a) R_{\rm h}^{-2}(a).} \tag{9}$$

Equivalently,

$$\Omega_{HM}(a) = \frac{\rho_{\rm HM}}{3M_{\rm Pl,red}^2 H^2} = \frac{\mu_{\rm h}(a)}{H^2(a)R_{\rm h}^2(a)}. \tag{10}$$

Remark 1 (Relation to the original $H^2\mu(a)$ form)**.** *The pre-Euclid note used $\rho_{\rm HM} = 3M_{\rm Pl,red}^2 H^2\mu(a)$ as a phenomenological Hubble-scale expression. Eq. (9) is the canonical v1.2 form. The original form is recovered when $R_{\rm h} \sim H^{-1}$, with the appropriate redefinition of the response factor.*

MINIMAL EQUATION OF STATE RELATION

If the horizon-maintenance component obeys an effective continuity equation,

$$\frac{d\ln\rho_{\rm HM}}{d\ln a} = -3(1 + w_{\rm HM}), \tag{11}$$

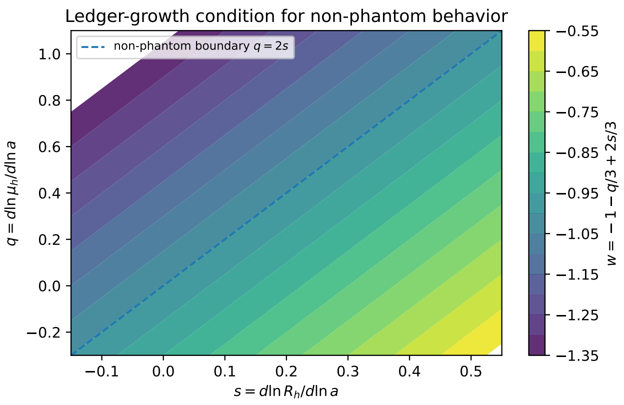

FIG. 2. Ledger-growth condition. With $s = d\ln R_h/d\ln a$ and $q = d\ln \mu_h/d\ln a$, the strict non-phantom boundary is $q = 2s$. Values above the dashed line are phantom in the effective physical sector.

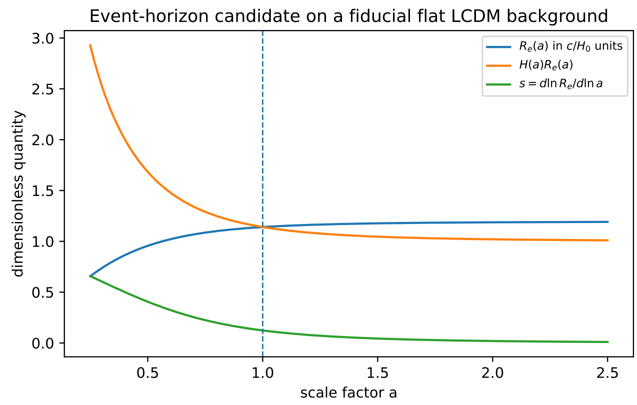

FIG. 3. Event-horizon candidate on a fiducial flat matter+ΛCDM background. The figure reports $R_e(a)$, $H(a)R_e(a)$, and $s = d\ln R_e/d\ln a$. This is a normal-form diagnostic, not a fit.

then Eq. (9) gives the central X1 relation:

Theorem 2 (Minimal $w(a)$ relation)**.** *For $\rho_{\rm HM} = 3M^2_{\rm Pl,red}\mu_{\rm h}R_{\rm h}^{-2}$ and an effective continuity equation, the equation of state is*

$$\boxed{w_{\rm HM}(a) = -1 - \frac{1}{3}\frac{d\ln \mu_{\rm h}}{d\ln a} + \frac{2}{3}\frac{d\ln R_{\rm h}}{d\ln a}.} \tag{12}$$

Criterion 1 (Non-phantom ledger-growth condition)**.** *The strict non-phantom X1 version, $w_{\rm HM} \geq -1$, is equivalent to*

$$\boxed{\frac{d\ln \mu_{\rm h}}{d\ln a} \leq 2\frac{d\ln R_{\rm h}}{d\ln a}.} \tag{13}$$

Thus a true physical phantom component corresponds to an occupancy response growing faster than the available horizon-area ledger.

Define

$$s(a) \equiv \frac{d\ln R_{\rm h}}{d\ln a}, \qquad q(a) \equiv \frac{d\ln \mu_{\rm h}}{d\ln a}. \tag{14}$$

Then $w_{\rm HM} = -1 - q/3 + 2s/3$. A useful response parametrization is

$$q(a) = 2\sigma(a)s(a), \tag{15}$$

which gives

$$w_{\rm HM}(a) = -1 + \frac{2}{3}[1 - \sigma(a)]s(a). \tag{16}$$

The strict version corresponds to $\sigma(a) \leq 1$ when $s(a) > 0$.

HORIZON CHOICE AND EVENT-HORIZON SPECIALIZATION

The main unresolved physical issue is the selection of $R_{\rm h}(a)$. Candidate choices include the apparent horizon, future event horizon, causal diamond, or a future FDS causal-access boundary defined by recordability and boundary-maintenance criteria. X1 v1.2 does not claim that this selection is solved.

For the future event horizon,

$$R_{\rm e}(t) = a(t)\int_t^\infty \frac{dt'}{a(t')}, \tag{17}$$

so

$$\dot{R}_{\rm e} = HR_{\rm e} - 1, \qquad \frac{d\ln R_{\rm e}}{d\ln a} = 1 - \frac{1}{HR_{\rm e}}. \tag{18}$$

Then

$$w_{\rm HM} = -1 - \frac{1}{3}\frac{d\ln \mu_{\rm h}}{d\ln a} + \frac{2}{3}\left(1 - \frac{1}{HR_{\rm e}}\right). \tag{19}$$

Using Eq. (10),

$$\frac{1}{HR_{\rm e}} = \sqrt{\frac{\Omega_{HM}}{\mu_{\rm h}}}, \tag{20}$$

so

$$w_{\rm HM} = -1 - \frac{1}{3}\frac{d\ln \mu_{\rm h}}{d\ln a} + \frac{2}{3}\left(1 - \sqrt{\frac{\Omega_{HM}}{\mu_{\rm h}}}\right). \tag{21}$$

RECONSTRUCTION PROTOCOL

Protocol 1 (Horizon-occupancy reconstruction). Given reconstructed $H(a)$, $\Omega_{DE}(a)$, and a candidate $R_{\rm h}(a)$, define the reconstructed occupancy

$$\boxed{\mu_{\rm h}^{\rm rec}(a) = \Omega_{DE}(a)H^2(a)R_{\rm h}^2(a).} \tag{22}$$

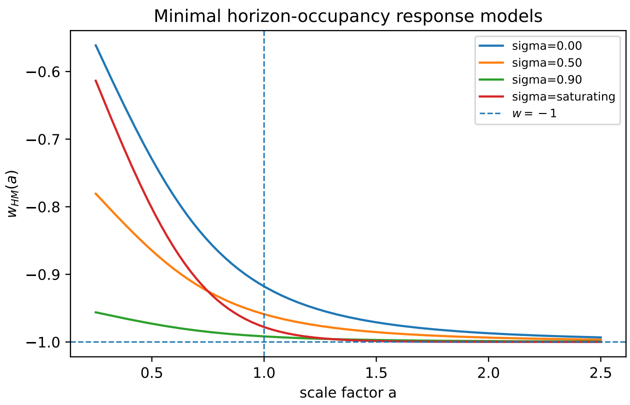

FIG. 4. Minimal response models using $d \ln \mu_h / d \ln a = 2\sigma d \ln R_h / d \ln a$ on the event-horizon candidate. The strict X1 version is non-phantom when the response stays within the ledger-growth bound.

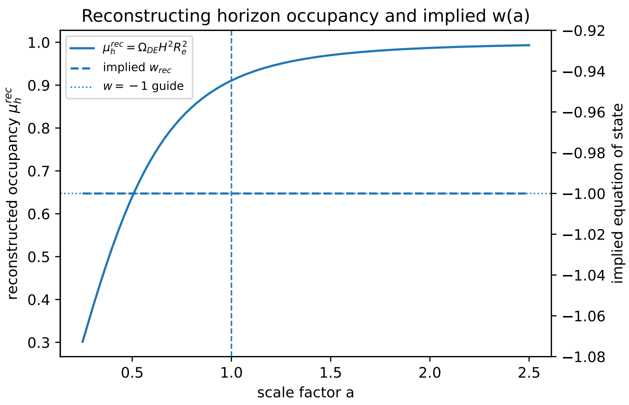

FIG. 5. Illustrative reconstruction protocol on a fiducial flat ΛCDM background with an event-horizon candidate. A strict X1 analysis would apply this diagnostic to observational reconstructions, not to this normal-form example alone.

Then compute the response diagnostic

$$\sigma_{\rm rec}(a) = \frac{d \ln \mu_{\rm h}^{\rm rec} / d \ln a}{2 d \ln R_{\rm h} / d \ln a} \qquad (23)$$

where the denominator is nonzero. X1 expects the reconstructed occupancy to be bounded, regular, horizon-tied, and non-phantom in the strict version.

PRE-REGISTERED OUTCOME TABLE

RELATION TO EXISTING MODELS

Relation to ΛCDM

ΛCDM is recovered as a limiting or demotion case when $\rho_{\rm DE}$ is constant and no meaningful dynamical occupancy is required. X1 does not reject ΛCDM by assumption. It asks whether future reconstructions reveal a bounded horizon occupancy with a nontrivial response law.

Relation to holographic dark energy

Holographic dark energy already motivates densities of the form $3c_{\rm HDE}^2 M_{\rm Pl,red}^2 L^{-2}$. X1 differs by interpreting the cutoff as a finite distinguishability boundary and by replacing a generic coefficient with an occupancy function tied to recordability, horizon selection, and demotion rules.

Relation to quintessence and modified gravity

Quintessence and modified gravity may produce effective $w(a)$ histories. X1 is not equivalent to either. It should be compared at the level of reconstructed density, non-phantom physical sector, perturbations, and whether $\mu_{\rm h}^{\rm rec}$ is regular and horizon-tied.

NUMERICAL NORMAL FORMS AND CODE

The accompanying code evaluates the equations above on a fiducial flat matter+ΛCDM background. It computes the event-horizon candidate,

$$R_{\rm e}(a) = a \int_a^\infty \frac{da'}{a'^2 E(a')}, \qquad (24)$$

constructs response models, produces $w(a)$ curves, reconstructs $\mu_{\rm h}^{\rm rec}$, and visualizes the non-phantom ledger condition. These figures are normal forms only. They are not a likelihood analysis and do not include perturbations.

LIMITATIONS AND NEXT WORK

The main limitations are explicit:

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1. X1 does not yet derive a unique $R_{\rm h}(a)$ selection rule.
2. X1 does not yet derive a microphysical response law for $\mu_{\rm h}(a)$.
3. X1 does not include perturbations, sound speed, clustering, or growth predictions.
4. X1 is not yet a likelihood-ready cosmological model.
5. X1 does not solve the microscopic vacuum-energy cancellation problem.

TABLE III. Pre-registered X1 outcome and demotion table.

Future outcome	**X1 status**	**Reason**
$\rho_{DE}(a)$ reconstructs as horizon-tied with bounded $\mu_h(a)$ and $w \geq -1$	Supports strict X1	Matches the density scale, occupancy protocol, and non-phantom ledger condition.
Dynamical $\rho_{DE}(a)$ appears but requires effective phantom crossing due to sector mixing or modified gravity mapping	Compatible only in effective sense	Strict physical component must remain non-phantom or be demoted.
High-precision data favor constant ρ_{DE} and no horizon response	Demotes dynamic X1	A static boundary interpretation may remain, but the dynamical claim weakens.
Data require a true fundamental phantom dark-energy density	Falsifies strict X1	Violates the ledger-growth condition if not attributable to effective-sector reconstruction.
No bounded or regular μ_h can be reconstructed for any physically justified horizon	Demotes or retires X1	The horizon-maintenance interpretation loses operational content.

The next step is therefore not to add more interpretation, but to derive or constrain $R_{\rm h}(a)$ and $\mu_{\rm h}(a)$ tightly enough to compete with ΛCDM, HDE, quintessence, and modified-gravity parameterizations.

CONCLUSION

FDS-X1 v1.2 integrates the pre-Euclid prediction registry with the horizon-ledger occupancy model. The result is a compact conditional claim: if dark energy is a horizon-maintenance component, then its density should be written as $3M_{\rm Pl,red}^2\mu_{\rm h}R_{\rm h}^{-2}$, its equation of state should obey Eq. (12), and the strict physical version should satisfy the non-phantom ledger-growth bound Eq. (13). The framework is falsifiable: constant-density data demote the dynamical version, true physical phantom behavior falsifies the strict version, and failure to reconstruct a bounded horizon occupancy weakens the bridge. X1 is now a constrained horizon-ledger reconstruction program rather than a bare scaling analogy.

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