Effective Geometry as Horizon Boundary Accounting: Finite Distinguishability, Horizon Entropy, and Thermodynamic Closure in Finite Distinction Systems

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General relativity describes gravity as spacetime geometry. Black-hole thermodynamics and Jacobson's local-horizon argument suggest that spacetime dynamics can also be read as an equation of state relating energy flux, horizon entropy, area variation, and causal structure. FDS-T2 develops this connection inside Finite Distinction Systems. It does not derive general relativity from FDS alone, replace Einstein gravity, derive quantum gravity, or derive the numerical coefficient in the Bekenstein-Hawking entropy formula. Instead, it formulates a boundary-thermodynamic bridge: when finite observers are constrained by horizon-bounded distinguishability budgets, effective geometry functions as the macroscopic accounting structure for causal access, horizon entropy, energy-momentum flux, and boundary-maintenance cost.

The novelty of T2 is not a new derivation of Einstein gravity, but an observer-relative reinterpretation of horizon thermodynamic variables as finite distinguishability ledgers: area counts accessible boundary distinctions, heat flux updates the ledger, and effective geometry is the covariant compression that preserves causal access and stress-energy accounting. T2 treats Jacobson-style local horizon thermodynamics as the central physical model class, not as an FDS theorem. The formal FDS contribution is the finite-boundary maintenance discipline: capacity, projection, update windows, resource ledgers, and invariant coarse variables. The refinements add covariant consistency for residual tensors, concrete horizon-task examples, and a clearer minimal-sufficient-complexity interpretation of geometry as a Phase-B boundary variable. The physical bridge assumptions are explicit: local causal horizons, area entropy, Unruh or surface-gravity temperature, Clausius-type horizon closure, and local covariance.

Reader contract. T2 is an effective-geometry bridge, not a replacement for general relativity. It does not claim that spacetime geometry is "nothing but information," does not derive the Einstein field equations from FDS alone, does not derive the Bekenstein-Hawking coefficient 1/4, and does not solve quantum gravity. It uses horizon thermodynamics as a physical bridge. If that bridge fails, the T2 interpretation is demoted; the formal FDS core remains unaffected.

Claim-status summary

Table I separates formal finite-system claims, standard physics inputs, and T2 bridge claims.

Keywords

Finite Distinction Systems; horizon thermodynamics; effective geometry; finite distinguishability; boundary accounting; Bekenstein bound; holography; Jacobson gravity; Unruh effect; black-hole thermodynamics; entropy production; causal horizons; Phase-B invariants; horizon-maintenance density.

INTRODUCTION

The geometry-thermodynamics problem

General relativity represents gravity through geometry,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{1}$$

Black-hole thermodynamics assigns entropy and temperature to horizons [13–15]. Jacobson's local-horizon argument showed that Einstein's equation can be obtained as an equation of state by applying the Clausius relation

$$\delta Q = T\,\delta S \tag{2}$$

to all local Rindler horizons, with entropy proportional to horizon area [18]. This suggests that geometry, entropy, and energy flux are not independent accounting languages.

T2 asks a narrower question than earlier geometry drafts: what is the FDS interpretation of effective geometry when finite observers have bounded causal access and horizons carry entropy? The answer proposed here is not that FDS derives GR. It is that effective geometry is the macroscopic boundary-thermodynamic accounting form for finite causal access, horizon entropy, and stress-energy flux.

TABLE I. Central T2 claims, status, and failure or demotion conditions.

Claim	Status	What would weaken or falsify it
Finite observers have bounded distinguishability budgets.	FDS / T1 bridge	A finite physical observer reliably registers, preserves, and updates unlimited distinctions with finite resources.
Horizons act as causal-access boundaries.	GR / QFTCS bridge	Local causal horizons are irrelevant to observable access, entropy, or thermodynamic accounting.
Horizon entropy gives boundary area accounting.	Physical bridge	Horizon entropy is shown not to scale with boundary area in the stated horizon class.
Clausius-type local horizon closure links heat flow and entropy variation.	Model-class bridge	Local horizon thermodynamics fails as an effective equation-of-state model.
Effective geometry can be read as boundary thermodynamic accounting.	Main T2 thesis	Geometry is shown to have no relation to causal access, horizon entropy, stress-energy flow, or boundary constraints.
Non-equilibrium horizon accounting may require residual terms.	Optional extension	All non-equilibrium horizon settings obey pure equilibrium closure with no entropy production, memory, noise, or correction terms.

What T2 adds beyond Jacobson

Jacobson's argument gives the standard horizon-thermodynamic model-class bridge: local Clausius closure plus area entropy yields Einstein-type geometry as an equilibrium equation of state. T2 adds three FDS-specific layers.

First, it gives an *observer-relative distinguishability interpretation*: S_H and A_H are read as a finite boundary ledger for accessible horizon distinctions, not merely as formal thermodynamic variables. Second, it introduces an auditable *boundary ledger formalism*, collecting H, A_H, S_H, T_H, δQ_H, τ, and residual error terms into a single accounting object. Third, it supplies an *overflow / Phase-B interpretation*: when microscopic horizon degrees of freedom are inaccessible, effective geometry is treated as a stable coarse boundary variable rather than a full microscopic ontology.

Thus the T2 contribution is

> Jacobson model-class bridge + finite distinguishability ledger interpretation, not FDS-derived GR.

Central thesis

The central thesis is

> Effective geometry is the covariant boundary ledger that closes horizon entropy, causal access, and stress-energy accounting for finite observers.

In symbols, the bridge is

$$(C_H, S_H, T_H, \delta Q_H) \implies g^{\text{eff}}_{\mu\nu} \text{ as boundary thermodynamic ledger.} \tag{3}$$

The arrow is not a formal derivation from FDS primitives. It is a physical bridge based on standard horizon-thermodynamic ingredients.

Why T2 follows T1, T3, P-series, and X3

T2 is downstream of safer finite-system modules. T1 defines finite observer distinguishability budgets and explicitly treats Bekenstein and holographic bounds as bridge inputs, not as FDS-derived theorems [2]. T3 analyzes overflow: when task-relevant distinction demand exceeds accessible capacity, the finite record may exit into coarse-graining, effective stochasticity, hysteresis, externalization, or Phase-B invariants [3]. P3, P4, P6, and P7 supply finite-bath memory, anti-recurrence, speed–precision–resource bounds, and invariant protection [4–7]. X3 places gravity in the four-operation closure as global boundary / causal geometry / stress-energy accounting [8]. T2 supplies a technical bridge behind that row.

FDS BACKGROUND

Active finite distinction systems

An active finite distinction system is represented by

$$\mathcal{S} = (X, E, B, M, Y, A, U, \pi, \ell, \Phi, \mathcal{P}, \tau), \tag{4}$$

where X is internal state, E environment, B boundary, M memory or model state, Y observation channel, A action space, U update rule, π finite projection, ℓ boundary-maintenance loss, Φ resource budget, $\mathcal{P}$ perturbation or pruning family, and τ update timescale [1]. T2 emphasizes B, Y, π, Φ, and τ: a causal boundary is not merely a surface but an access, update, and accounting interface.

Finite distinguishability budget

For a finite observer O with finite projection π_O, T1 defines

$$N_O = |\mathrm{Im}(\pi_O)|, \qquad C_O = \log_2 N_O. \tag{5}$$

Accessible capacity is bottlenecked by memory, boundary access, channel capacity, causal reach, externalized capacity, and update throughput [2]. T2 uses this language to read a horizon as the boundary of causal access, not as an all-purpose proof that geometry is information.

Overflow and Phase-B variables

T3 defines an overflow deficit of the form

$$\Delta(t) = R^{(\tau)}_{\min}(\varepsilon; \Psi_t) - C_{\mathrm{acc}}(t). \tag{6}$$

When $\Delta > 0$, microstate detail cannot be tracked at full fidelity. The accessible description may become stochastic, hysteretic, or dominated by coarse variables. T2 treats effective geometry as a candidate Phase-B boundary variable: a stable coarse structure that tracks causal access, horizon area, and stress-energy flow after microscopic horizon degrees of freedom are inaccessible.

STANDARD PHYSICS BASELINE

Horizon entropy and temperature

For black-hole horizons, the entropy is

$$S_{\mathrm{BH}} = \frac{k_B A}{4\ell_P^2}, \qquad \ell_P = \sqrt{\frac{\hbar G}{c^3}}. \tag{7}$$

T2 treats this relation as established black-hole thermodynamics and holographic boundary accounting, not as something derived here.

For a local accelerated observer, Unruh temperature is

$$T_U = \frac{\hbar a}{2\pi k_B c}, \tag{8}$$

and for a stationary horizon with surface gravity κ,

$$T_H = \frac{\hbar \kappa}{2\pi k_B c}. \tag{9}$$

These relations provide the temperature bridge between energy flux and horizon entropy.

Jacobson bridge

In Jacobson's model class, the heat flux through a local Rindler horizon is related to stress-energy,

$$\delta Q = \int T_{\mu\nu} \chi^\mu d\Sigma^\nu, \tag{10}$$

while the entropy variation is proportional to area,

$$\delta S = \eta\, \delta A. \tag{11}$$

Imposing $\delta Q = T\delta S$ for all local horizons yields the Einstein equation as an equation of state, with the coupling fixed when $\eta = k_B/(4\ell_P^2)$ [18]. T2 does not reproduce the full derivation. It uses it as the central bridge showing how local horizon thermodynamics can encode effective geometry.

DEFINITIONS

Definition 1 (Finite horizon boundary)**.** *A finite horizon boundary H is a causal boundary relative to an observer or finite record-bearing system O, delimiting the region whose events can be operationally distinguished, updated, or included in the observer's boundary-maintenance ledger over a finite window τ.*

Definition 2 (Horizon distinguishability budget)**.** *The horizon distinguishability budget is*

$$C_H = \frac{S_H}{k_B \ln 2}. \tag{12}$$

For an area-law horizon,

$$C_H = \frac{A_H}{4\ell_P^2 \ln 2}. \tag{13}$$

This is a bridge expression: it imports horizon entropy as an area ledger and expresses it as accessible distinguishability.

Definition 3 (Boundary thermodynamic ledger)**.** *A boundary thermodynamic ledger is a tuple*

$$\mathcal{L}_H = (H, A_H, S_H, T_H, \delta Q_H, \tau, \mathcal{E}_H), \tag{14}$$

where H is a causal or horizon boundary, A_H its area, S_H its entropy, T_H its temperature, δQ_H the energy flux or heat flow assigned to the boundary, τ the operational update window, and $\mathcal{E}_H$ an admissible coarse-grained error or non-equilibrium term.

Definition 4 (Effective geometry)**.** *An effective geometry is a metric, connection, or causal structure $g^{\mathrm{eff}}_{\mu\nu}$ that compresses the boundary ledger into a stable macroscopic relation between causal access and stress-energy flow:*

$$g^{\mathrm{eff}}_{\mu\nu} = \mathcal{G}(\mathcal{L}_H). \tag{15}$$

This definition is operational. It does not assert that the metric is the microscopic substrate of spacetime.

Definition 5 (Admissible ledger-to-geometry map)**.** *The map $\mathcal{G} : \mathcal{L}_H \mapsto g^{\mathrm{eff}}_{\mu\nu}$ is admissible only if it preserves, to registered tolerance ε, (i) causal ordering and light-cone* *structure, (ii) the horizon-area functional, (iii) the response of area variation to local stress-energy flux, (iv) local covariance, (v) closure residuals, and (vi) coarse-grained stability under the registered perturbation family. Thus $\mathcal{G}$ is not an arbitrary relabeling; it is a constrained compression from a horizon ledger to an effective geometric structure.*

Definition 6 (Boundary-thermodynamic closure)**.** *A horizon ledger is thermodynamically closed to tolerance ε if*

$$|\delta Q_H - T_H \delta S_H - \delta W_H - \delta \Sigma_H| \leq \varepsilon, \tag{16}$$

where δW_H is a work-like boundary term and $\delta \Sigma_H$ is an entropy production, memory, or non-equilibrium correction. Equilibrium Jacobson closure is the special case $\delta W_H \approx 0$ and $\delta \Sigma_H \approx 0$.

MAIN PROPOSITIONS

Proposition 1 (Jacobson model-class bridge)**.** *Assume horizon entropy $S_H = \eta A_H$, local Unruh or surface-gravity temperature, Clausius closure $\delta Q = T_H \delta S_H$, and local covariance for all admissible local causal horizons. Then an Einstein-type geometric equation arises as an equilibrium equation of state in the Jacobson model class.*

Remark 1. *This proposition is a standard physics bridge, not an FDS theorem. T2 inherits it as a model-class result from horizon thermodynamics.*

Proposition 2 (FDS boundary-ledger interpretation)**.** *If the Jacobson model-class bridge holds, then FDS interprets $g_{\mu\nu}^{\mathrm{eff}}$ as the stable macroscopic compression of a finite horizon distinguishability ledger. Horizon area counts accessible boundary distinctions; heat flux updates the ledger; and effective geometry preserves causal access and stress-energy accounting in covariant form.*

Proof sketch. Finite horizon area bounds accessible distinguishability through $C_H \sim A_H/\ell_P^2$. Horizon entropy is the logarithm of accessible boundary state count, $S_H = k_B \ln N_H$. Energy flux through the boundary changes the horizon ledger. Local temperature converts entropy variation into heat flow, $\delta Q_H = T_H \delta S_H$, at equilibrium. Requiring this relation for all local causal horizons forces a covariant relation between stress-energy and local geometric response. Jacobson's theorem supplies the equilibrium model-class result. T2 adds the FDS interpretation: the metric is the stable macroscopic boundary ledger for finite causal access and stress-energy flow. □

Remark 2 (What the propositions do not prove)**.** *These propositions do not derive the Bekenstein-Hawking coefficient, the Unruh effect, local Lorentz invariance, quantum field theory in curved spacetime, or general relativity from FDS alone. The structure is*

$$\text{standard physics bridge+FDS ledger interpretation} \neq \text{FDS deriv} \tag{17}$$

The failure of any physical bridge assumption demotes the T2 interpretation but not the FDS finite-capacity core.

WHY GEOMETRY?

Causal access is geometric

Finite observers have memory and channel limits, but also causal-access limits. Over an operational window, accessible distinctions lie inside a causal region such as a causal diamond. Geometry encodes light cones, time ordering, horizons, area of access surfaces, and the relation between energy-momentum and causal response. This makes geometry a natural representation for boundary accessibility.

Area as boundary accounting

In horizon thermodynamics, area is not merely shape. It is an entropy ledger:

$$S_H \propto A_H. \tag{18}$$

T2 reads area as the macroscopic counter of boundary-accessible distinguishability. This is not a claim that every surface carries maximal entropy; it is a statement about the horizon or covariant-boundary class where area entropy is physically established.

Curvature as ledger response

Energy flux through a local horizon changes the boundary ledger. The effective response is focusing, area variation, and curvature. In the T2 interpretation,

> Curvature is the covariant way a finite causal boundary records stress-energy flux.

This is interpretive language for the Jacobson model class, not a new gravitational field equation.

RELATION TO JACOBSON GRAVITY

Table II summarizes the mapping. T2 inherits Jacobson's strength and limitations. If local horizon thermodynamics is an accurate equation-of-state model, then geometry has a thermodynamic reading. If it fails, the T2 bridge fails at exactly that layer.

NON-EQUILIBRIUM EXTENSION

Horizon capacity deficit

For a horizon-boundary task family Ψ_H, define

$$\Delta_H(\tau) = R^{(\tau)}_{\min}(\varepsilon; \Psi_H) - C_H. \tag{19}$$

Examples of Ψ_H include task families for local horizon-area variation, stress-energy flux records, causal-diamond boundary updates, and coarse records of unresolved horizon microstates. When $\Delta_H > 0$, the boundary ledger cannot track all task-relevant horizon distinctions at full fidelity over the update window. The missing distinctions may enter the effective description as entropy production, memory, stochastic noise, hysteresis, or coarse correction terms.

Ledger residuals

A non-equilibrium ledger may be written in audit form as

$$\delta Q_H = T_H \delta S_H + \delta \Sigma_H + \delta W_H, \tag{20}$$

or equivalently

$$R_H = \delta Q_H - T_H \delta S_H - \delta W_H - \delta \Sigma_H. \tag{21}$$

Here R_H measures closure failure after registered work-like and entropy-production terms are included.

A covariant effective description may place such failures into a ledger-residual slot,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + R^{\text{ledger}}_{\mu\nu}, \tag{22}$$

with the bookkeeping decomposition

$$R^{\text{ledger}}_{\mu\nu} = R^{\text{prod}}_{\mu\nu} + R^{\text{mem}}_{\mu\nu} + R^{\text{noise}}_{\mu\nu} + R^{\text{curv}}_{\mu\nu} + R^{\text{hyst}}_{\mu\nu}. \tag{23}$$

This is not proposed as a new gravitational field equation. It is a ledger-location marker for non-equilibrium residuals: entropy production, memory kernels, unresolved boundary noise, higher-curvature slots, and hysteretic response.

TABLE II. Jacobson inputs and T2 interpretation.

Jacobson input	T2 interpretation
local Rindler horizon	finite causal boundary
horizon area A	distinguishability ledger
$\delta S = \eta \delta A$	boundary-state accounting
δQ	stress-energy flux across boundary
T	flux-to-entropy conversion scale
Einstein equation	equilibrium closure for all ledgers

Any promoted residual tensor must also satisfy the corresponding covariant consistency condition. If the matter stress-energy tensor is separately conserved, then

$$\nabla^\mu R^{\text{ledger}}_{\mu\nu} = 0. \tag{24}$$

If the effective matter sector is not separately closed, the residual must instead enter a balanced exchange equation,

$$\nabla^\mu \left(\frac{8\pi G}{c^4} T_{\mu\nu} + R^{\text{ledger}}_{\mu\nu} \right) = 0. \tag{25}$$

This condition is required by the Bianchi identity and local covariance; without it the residual slot is not an admissible geometric accounting term.

Recent analyses of non-equilibrium horizon thermodynamics in modified gravity emphasize that entropy-production terms can have different origins in local Rindler and cosmological apparent-horizon frameworks, and that covariant consistency conditions such as the Bianchi identity matter for any promoted residual term [26].

T2 does not claim that the fundamental Newton constant varies with observer capacity. Capacity deficit enters the effective description through closure residuals, memory terms, stochastic response, or loss of full-fidelity horizon accounting, not through a bare replacement $G \to G_{\text{eff}}(C_{\text{acc}})$.

Connection to non-equilibrium gravity

Non-equilibrium horizon thermodynamics has been used to motivate entropy production terms and modified gravitational dynamics [21]. T2 is compatible with this direction, but keeps the claim weaker: non-equilibrium boundary accounting should appear as residual terms, not necessarily as one universal modification of GR.

GEOMETRY AS A PHASE-B BOUNDARY VARIABLE

T3 describes Phase-B variables as coarse variables that remain cheap to update, slow to forget, and approximately closed after microscopic tracking fails. In T2, effective geometry can be modeled as a minimal sufficient boundary variable. A normal-form selection score is

$$q^\star_{\text{geom}} = \arg \min_{q \in \mathcal{Q}_{\text{geom}}} \left[C_{\text{update}}(q) + \lambda M_{\text{closure}}(q) + \mu \ell_{\text{pred}}(q) \right]. \tag{26}$$

Here C_{update} is the cost of maintaining and updating the coarse variable, M_{closure} is the residual closure or Markovianity error, and ℓ_{pred} is the loss for predicting causal access, horizon-area response, or stress-energy flux. This is not a gravitational variational principle. It is a finite-system selection principle: geometry survives overflow because it is a minimal sufficient boundary variable for causal access and stress-energy accounting.

Minimal sufficient complexity

A microscopic boundary description may carry more detail than a finite observer can update or verify. Among possible compressed boundary variables, a geometry-like variable is selected when it is sufficient for the registered task class: it preserves causal order, predicts area response, and tracks stress-energy flux to tolerance while requiring less update and closure cost than micro-boundary tracking. In T2 language, effective geometry is not the most detailed description; it is the minimal sufficient covariant description that remains usable after horizon capacity overflow.

Optional bridge: information geometry

Information geometry provides a related but non-identical example of how distinguishability can induce geometry [25]. Fisher-information metrics define distances on statistical manifolds by the local distinguishability of probability distributions. T2 does not identify spacetime geometry with Fisher geometry. It uses the analogy only to mark a possible bridge: when distinguishability relations are compressed into stable differential structure, metric language can emerge as the natural accounting form.

RELATION TO T1, T3, P-SERIES, AND X3

T1 supplies the budget language. T3 supplies the overflow language: when micro-boundary distinctions exceed accessible capacity, coarse stochastic variables and Phase-B invariants dominate. P3 explains how inaccessible environmental memory produces effective finite-bath terms. P4 explains why lost preimage information is not recovered by capacity restoration alone. P6 supplies update-rate and dissipation bounds. P7 supplies invariant-ledger logic. X3 states gravity's functional role; T2 explains that role through horizon thermodynamic accounting.

COROLLARY: HORIZON-MAINTENANCE DENSITY SCALE

T2 does not derive dark energy and does not identify any horizon scale with the observed cosmological constant. However, once horizon entropy and temperature are treated as a boundary ledger, a natural horizon-scale energy estimate appears:

$$E_H \sim T_H S_H. \tag{27}$$

Distributed over a horizon volume $V_H \sim R_H^3$, this gives the dimensional density scale

$$\rho_H \sim \frac{E_H}{V_H} \sim \frac{c^4}{G R_H^2} \tag{28}$$

up to convention-dependent numerical factors. T2 only identifies the upstream ledger structure. The cosmological interpretation of this scale is developed separately in FDS-X1. The horizon-ledger interpretation is naturally compatible with non-phantom maintenance-density readings because a stable causal ledger should not require indefinite violation of causal-order accounting, but the equation-of-state claim is not made in T2.

Recent work on dynamical de Sitter horizons and de Sitter first-law thermodynamics reinforces the need to distinguish stationary horizon entropy from non-stationary horizon-ledger accounting [27, 29].

EMERGENT GRAVITY AND QUANTUM-GRAVITY CAVEATS

T2 is compatible with thermodynamic or emergent readings of gravity, but it does not require a completed microscopic theory of spacetime. It also does not settle whether gravitons are fundamental or emergent. A cautious statement is: if geometry is an effective thermodynamic field, perturbative gravitons may be interpreted as quanta of the effective field in regimes where linearized quantization is valid. T2 does not decide the ontology of the graviton.

Similarly, T2 does not solve quantum gravity. It supplies a bridge between finite distinguishability, horizon entropy, and effective geometry. A microscopic theory of the horizon degrees of freedom is outside the scope of this paper.

NUMERICAL AND CONCEPTUAL DEMONSTRATIONS

The figures are deterministic normal-form demonstrations generated by `code/generate_results.py`. They are not empirical fits and not simulations of full general relativity.

TABLE III. Relation of T2 to neighboring FDS papers.

Paper	**Core role**	**T2 relation**
FDS Core	finite capacity, physical bridge discipline	formal foundation for boundary accounting [1]
T1	finite distinguishability budgets	upstream budget and horizon-capacity language [2]
T3	overflow and effective stochasticity	geometry as Phase-B boundary variable [3]
P3	finite-bath memory	horizon microstates as inaccessible bath analogue [4]
P4	anti-recurrence and hysteresis	capacity recovery does not recover erased boundary microstates [5]
P6	speed, precision, and dissipation	horizon update windows and throughput accounting [6]
P7	invariant protection	geometry as protected large-scale boundary ledger [7]
X3	gravity as boundary / geometry / accounting	functional placement of the T2 bridge [8]
X1	horizon-maintenance density	downstream cosmological scale bridge [9]
T2	horizon boundary thermodynamics	effective geometry as boundary ledger

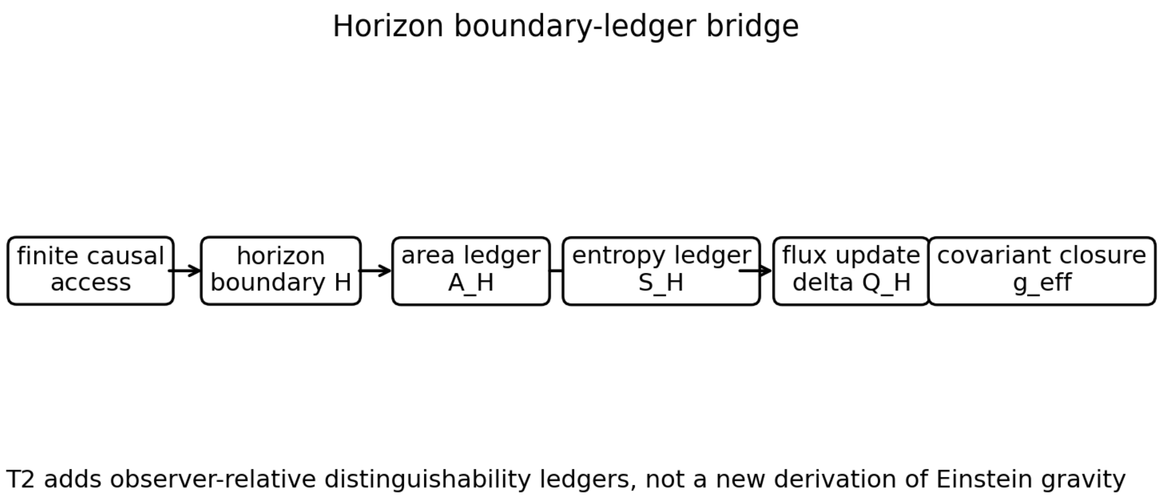

FIG. 1. Horizon boundary-ledger bridge. Finite causal access defines a horizon boundary; area functions as a ledger; entropy and flux closure provide the thermodynamic accounting; effective geometry is the macroscopic representation of that accounting.

TABLE IV. Non-equilibrium horizon-ledger residual taxonomy. This table replaces arbitrary toy bar weights with residual classes, ledger sources, and possible effective signatures. It is a placement guide for corrections, not a field equation.

Residual class	Ledger source	Possible effective signature
entropy production	$d_i S_H$	viscosity / dissipation
memory kernel	$K(t-t')$	nonlocal delayed response
stochastic noise	unresolved boundary modes	metric fluctuation proxy
curvature correction	high-curvature ledger terms	R^2 or $f(R)$-like slot
hysteresis	path-dependent boundary state	delayed area response

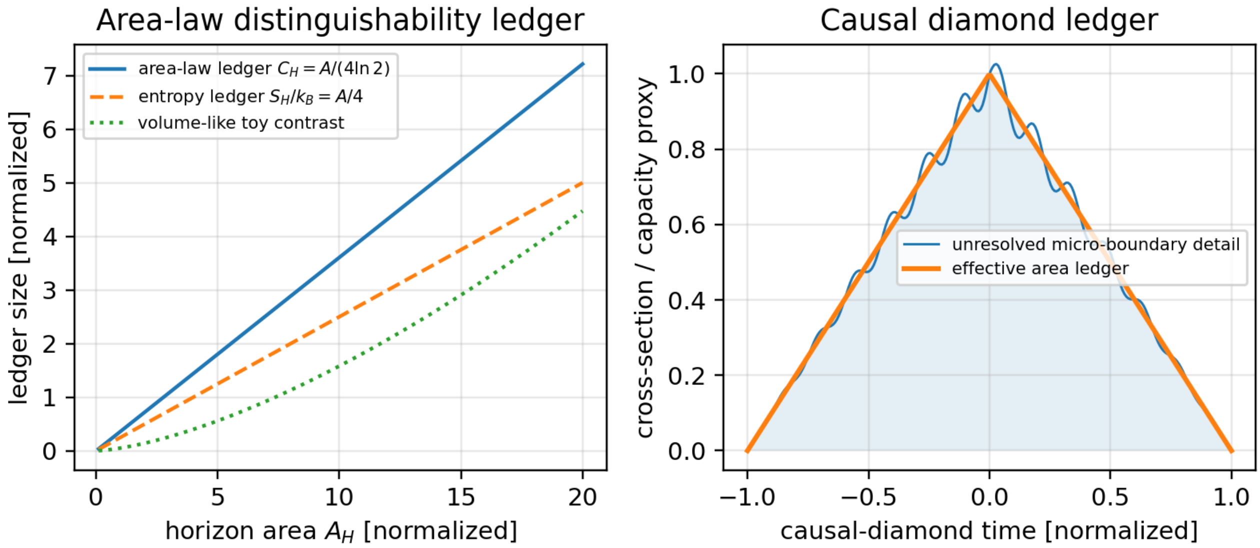

FIG. 2. Left: area-law horizon distinguishability ledger in normalized Planck-area units. The plot illustrates the bridge expression $C_H = A/(4\ell_P^2 \ln 2)$; it does not derive the coefficient. Right: causal-diamond normal form. An all-detail description contains unresolved boundary ripples, while the finite observer maintains a smooth effective area ledger. This is a conceptual illustration of effective geometry as coarse boundary accounting, not a metric solution.

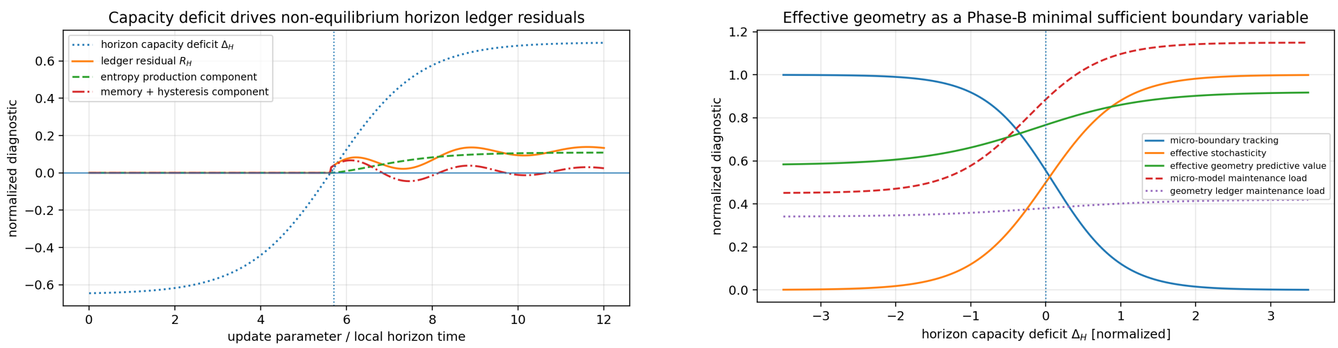

FIG. 3. Left: horizon capacity deficit Δ_H drives non-equilibrium ledger residuals in the normal form. Right: effective geometry as a Phase-B minimal sufficient boundary variable. After overflow, micro-boundary tracking falls and effective stochasticity rises, while coarse geometry retains predictive value at lower maintenance load.

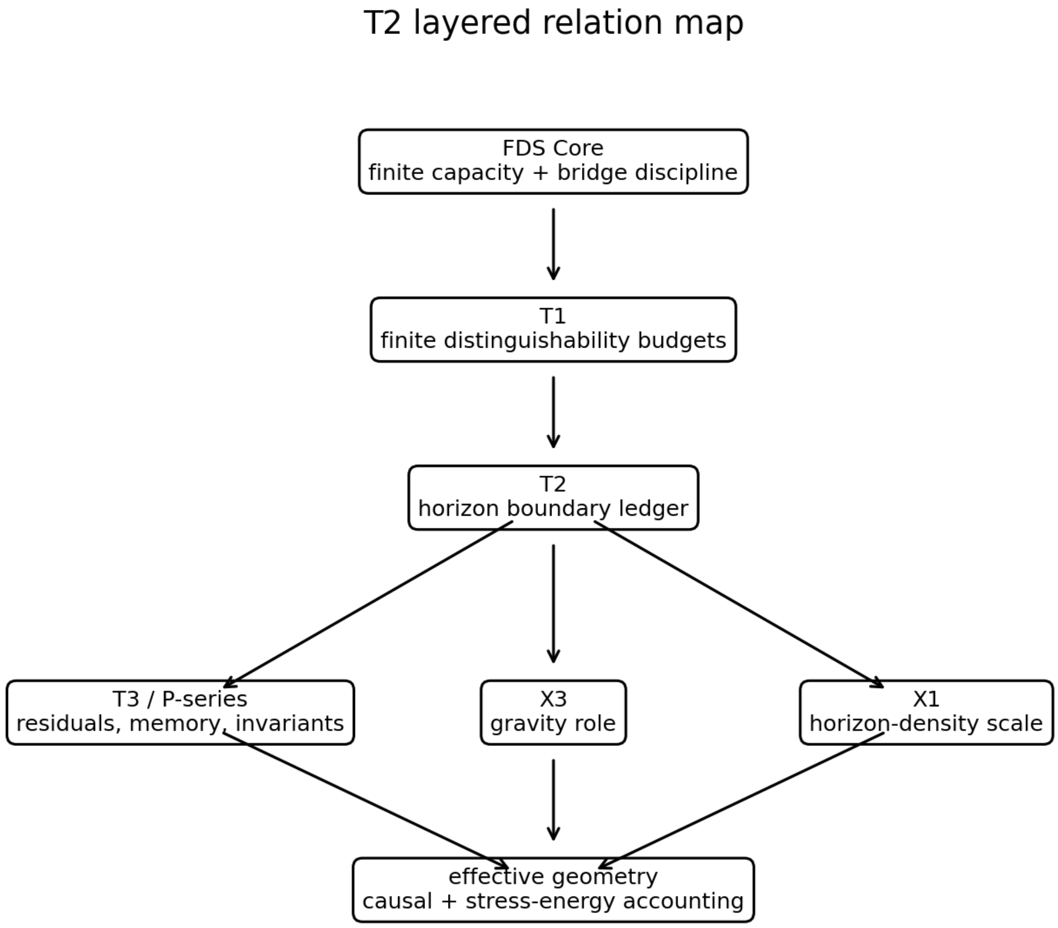

FIG. 4. Relation map. The layout makes the dependency stack explicit: FDS Core leads to T1 finite budgets and T2 horizon ledger, which branches to T3/P-series residual mechanisms, the X3 gravity role, and the X1 horizon-density scale.

FALSIFICATION AND DEMOTION CONDITIONS

T2 is weakened or demoted if horizon entropy is not related to boundary area for the stated horizon class, local horizon temperature fails in the required domain, Clausius-style horizon thermodynamics fails as an effective equation-of-state model, stress-energy flux cannot be related to causal-boundary area change even approximately, geometry has no operational relation to causal access or boundary entropy, or non-equilibrium corrections cannot be represented as ledger residuals.

A strong demotion would be a complete physical theory in which geometry exists and evolves independently of any causal boundary, entropy, or energy-momentum accounting. Even then, the formal FDS finite-capacity results remain separate: only the T2 physical bridge is affected.

RELATION TO EXISTING THEORY

T2 uses standard physics as bridge input. General relativity provides the target effective geometry [10–12, 24]. Black-hole thermodynamics provides entropy and temperature [13–15]. Bekenstein and holographic bounds supply finite-boundary accounting [16, 17]. Jacobson's 1995 argument supplies the central local-horizon model class [18]. The Unruh effect supplies the local acceleration temperature [19]. Non-equilibrium thermodynamics supplies entropy-production language [20, 21]. Emergent-gravity approaches provide comparison points, not dependencies [22, 23]. Information geometry supplies an optional bridge between distinguishability and metric structure [25]. Because horizon-entropy functionals are actively generalized in black-hole and cosmological settings, T2 treats the area law and its coefficient as physical bridge inputs rather than as FDS-derived constants [28]. FDS supplies the finite-system bookkeeping and failure-propagation discipline [1–3].

CONCLUSION

T2 does not derive general relativity from FDS and does not replace the Einstein equation. It interprets the thermodynamic side of gravitational physics through finite distinction systems. A finite observer has bounded causal access and bounded distinguishability. A horizon is the causal boundary of that access. Horizon area functions as an entropy ledger; heat flux across the boundary changes that ledger; and requiring the ledger to close locally and covariantly yields effective geometric dynamics in the Jacobson model class.

Geometry is therefore not merely a coordinate system in T2. It is the stable macroscopic bookkeeping structure through which finite systems account for causal access, boundary entropy, and stress-energy flow. Effective geometry is horizon boundary thermodynamics written in covariant form.

APPENDIX A. OPERATIONAL ANALOGY FOR EMBODIED AGENTS

This appendix is not part of the physics proof. It extracts an engineering analogy: an embodied agent should treat geometry not only as coordinates, but as a boundary ledger for causal reach, uncertainty, and update cost.

TABLE V. T2 operational analogy for embodied agents.

Horizon geometry	**Agent analogue**
causal horizon	sensor / action reach
horizon area	interface bandwidth
entropy ledger	reachable-state uncertainty
energy flux	boundary update cost
curvature	world-model deformation
closure residual	latency, memory, drift, error

For autonomous driving or embodied robotics, when environmental complexity exceeds the sensor-processing boundary, the agent must geometricize: it must maintain a compressed reachability and uncertainty ledger rather than raw scene detail. Failure modes include mapping without reachability accounting, planning beyond actuation horizon, ignoring uncertainty growth at the boundary, assigning no resource cost to geometry updates, and brittle behavior under non-equilibrium dynamics. This is an analogy to T2 ledger logic, not a test of gravitational horizon thermodynamics.

Protocol 1 (Geometric boundary-ledger planning). *An embodied agent facing high-dimensional sensory overflow should: (i) estimate causal reach and sensor–action horizon; (ii) estimate boundary bandwidth and update budget; (iii) compress perception into a reachability and obstacle geometry; (iv) track uncertainty growth near the boundary of access; and (v) slow down, externalize, or replan when the agent-level capacity deficit becomes positive.*

The protocol is an engineering analogue of horizon-ledger accounting: geometry is a survival-oriented compression of causal reach, not merely a coordinate map.

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