Finite Distinguishability Budgets and Maintenance Bounds for Physical Observers

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A finite physical observer cannot register, preserve, update, and operationally use an unlimited number of distinctions. This paper defines an observer-relative distinguishability budget $N_{\mathcal{O}} = |\operatorname{Im}(\pi_{\mathcal{O}})|$ and bit capacity $C_{\mathcal{O}} = \log_2 N_{\mathcal{O}}$ for finite record-bearing systems. Using established Bekenstein, holographic, channel, and Landauer-style update constraints as physical bridge inputs, we formulate accessible distinguishability as a bottleneck over internal memory, boundary access, communication channel, causal reach, and irreversible update throughput. We then define boundary-relative capacity deficit by comparing task-relevant rate-distortion demand $R^{(\tau)}_{\min}(\varepsilon; \Psi)$ with accessible capacity. When demand persistently exceeds accessible budget, a finite observer must coarse-grain, compress, externalize, prune, relax the task, or fail. The paper does not derive gravitational entropy bounds, quantum gravity, the Standard Model, the Einstein equations, or the numerical coefficient in the Bekenstein-Hawking entropy formula. Its purpose is narrower: to make finite distinguishability a precise, auditable, and testable bridge between finite-system theory, information bounds, and physical observation.

Reader Contract. This paper does not claim to derive the Bekenstein bound, holography, Landauer's principle, the Einstein equations, quantum gravity, the Standard Model, or the numerical coefficient 1/4 in the Bekenstein-Hawking entropy formula. It uses established entropy and thermodynamic constraints as physical bridge inputs. Its contribution is to express them as observer-relative distinguishability budgets for finite record-bearing systems and to derive maintenance consequences when task demand exceeds accessible capacity.

Claim-status summary

Keywords: finite distinguishability; holographic bound; Bekenstein bound; observer-relative entropy; active finite distinction systems; boundary maintenance; information bounds; causal horizons; Landauer principle; finite observers.

INTRODUCTION

The finite-observer problem

Physical theories often describe states as if all degrees of freedom in a region were simultaneously available to an ideal observer. Operational physics does not work that way. A real observer is a finite physical system. It has finite memory, finite energy, finite update rate, finite causal access, finite error tolerance, finite record stability, and a finite boundary through which information must pass. Even when the underlying mathematical state space is large, the number of distinctions that can be registered and used by a finite system is limited.

Main concept

The central object of this paper is an observer-relative distinguishability budget. A finite system does not access the full physical state space directly. It resolves a space of possibilities into operationally distinguishable equivalence classes via a finite distinction projection $\pi_{\mathcal{O}}$:

$$N_{\mathcal{O}} = |\operatorname{Im}(\pi_{\mathcal{O}})|, \qquad C_{\mathcal{O}} = \log_2 N_{\mathcal{O}}, \tag{1}$$

where $\pi_{\mathcal{O}}$ is the projection by which $\mathcal{O}$, a finite record-bearing system, resolves a physical region into operationally distinguishable alternatives.

Why existing bounds matter

The Bekenstein bound limits the information that can be contained in a finite region of given energy and size [10]. The holographic principle asserts that the maximum entropy associated with a region scales with boundary area rather than volume [12–14]. Black-hole thermodynamics gives the limiting entropy

$$S_{\mathrm{BH}} = \frac{k_{\mathrm{B}} A}{4\ell_{\mathrm{P}}^2}, \tag{2}$$

where A is horizon area and $\ell_{\mathrm{P}} = \sqrt{\hbar G/c^3}$ is the Planck length [8, 9]. The present paper does not re-derive or modify these bounds. It treats them as physical bridge inputs that constrain the distinguishability of any finite record-bearing system.

TABLE I. Summary of central FDS-T1 claims, their epistemic status, and what would constitute falsification or demotion.

Claim	Status	What would fail
Finite record carrier → finite operational distinguishability	Operational / formal bridge	Bounded physical observer with unlimited reliable distinctions
Bekenstein ceiling on distinguishability	Physical bridge	Applicable bounded system exceeds Bekenstein information limit
Holographic ceiling	Physical bridge	Horizon-bounded system exceeds area information accounting
Accessible capacity bottleneck	Conditional theorem	Task uses distinctions that bypass all bottlenecks
Boundary-relative capacity deficit	Definition	Not falsified directly; only usefulness can fail
Budget-exit theorem	Conditional theorem	Full-fidelity maintenance under sustained deficit without exit
Maintenance inequality	Conditional physical bridge	Reliable irreversible erasure below Landauer cost under stated conditions
Budget crossing signature	Testable prediction	No transition in error, latency, compression, externalization, pruning, reset/erasure, heat, task relaxation, or failure under controlled crossing

Contributions

This paper makes nine contributions.

1. It defines an observer-relative finite distinguishability budget $N_{\mathcal{O}} = |\mathrm{Im}(\pi_{\mathcal{O}})|$ and bit capacity $C_{\mathcal{O}} = \log_2 N_{\mathcal{O}}$.
2. It states conservative physical bridges from finite record carriers to Bekenstein and holographic information ceilings.
3. It distinguishes stock capacity (storage/boundary/channel/memory) from update throughput (Landauer-limited irreversible operations).
4. It formulates accessible capacity as the minimum of stock and throughput.
5. It defines boundary-relative capacity deficit by comparing rate-distortion task demand against accessible capacity.
6. It derives a budget-exit classification: coarse-graining or compression, externalization, pruning, task relaxation, or failure when demand exceeds accessible budget.
7. It provides a maintenance inequality connecting positive deficit to Landauer-style thermodynamic lower bounds.
8. It makes budget crossing the central testable prediction, with explicit observables and test protocols.
9. It gives falsification conditions and a reproducibility checklist.

What is not claimed

This paper does not derive the Bekenstein bound, holography, Landauer's principle, the Einstein equations, quantum gravity, the Standard Model, or the numerical coefficient 1/4 in the Bekenstein-Hawking entropy formula. It uses established entropy and thermodynamic constraints as physical bridge inputs. Its contribution is to express them as observer-relative distinguishability budgets for finite record-bearing systems and to derive maintenance consequences when task demand exceeds accessible capacity.

Relation to companion papers

This paper is the physical finite-observer bridge in the FDS sequence. It depends on the formal FDS core [1] and the DT archive [2] for foundational definitions. It does not duplicate the downstream applications already developed in companion papers. Artificial agency is treated in the FDS-AI Agency paper [3]; active pruning in protocell-like systems is treated in FDS-L1 [4]; reportable cognitive access is treated in FDS-C1 [6]; and life/cognitive bridge claims are registered in FDS-LC0 [7]. The present paper supplies the shared finite-observer budget language used by those papers: observer-relative finite distinction projections, accessible capacity, boundary-relative capacity deficit, maintenance cost, and budget-exit classification.

RELATED WORK

Bekenstein bounds and holography

Bekenstein proposed a universal upper bound on the entropy-to-energy ratio for bounded systems [10]. In one common form, a system of energy E confined to a sphere of radius R satisfies

$$S \leq \frac{2\pi k_{\mathrm{B}} E R}{\hbar c}. \tag{3}$$

Measured in bits, this becomes

$$I \leq \frac{2\pi E R}{\hbar c \ln 2}. \tag{4}$$

The holographic principle strengthens the geometric intuition: the maximum entropy associated with a spatial region is controlled by the area of an appropriate boundary, not by the enclosed volume [12–14]. Bousso's covariant entropy bound generalizes the area law to light-sheets and avoids some limitations of simple spatial-volume statements [14].

The present paper does not re-prove these bounds. It treats them as physical constraints that any finite distinguishability theory must respect. Its novelty is to place them inside the FDS architecture: entropy bounds become upper limits on the number of distinctions that a finite boundary-maintaining system can operationally resolve, preserve, or use.

Black-hole thermodynamics and horizon entropy

Black-hole thermodynamics established that horizons carry entropy and temperature [8, 9]. The entropy formula (2) is the paradigmatic area law. Jacobson's thermodynamic derivation of the Einstein equation from local horizon thermodynamics suggests a deep connection between area, entropy, energy flux, and spacetime geometry [15]. Verlinde and related emergent-gravity approaches further explore the possibility that gravity is connected to information, entropy, and coarse-graining [16].

FDS-T1 remains upstream of a full gravity derivation. It does not infer Einstein equations. It supplies a finite-system reading of horizon entropy: a horizon is not only a geometric surface but also a boundary of finite distinguishability access for a class of observers.

Landauer erasure and information thermodynamics

Landauer's principle links logically irreversible information erasure to a thermodynamic heat cost [17]. Bennett clarified how reversible computation can avoid erasure costs locally while still requiring attention to garbage, memory reuse, and thermodynamic accounting [18]. Experiments have verified Landauer-type erasure costs in controlled systems [19–21]. Stochastic thermodynamics and feedback thermodynamics generalize these links in nonequilibrium systems [22–24].

FDS uses Landauer conservatively. It does not assert that every representation or inference step dissipates $k_{\mathrm{B}} T \ln 2$ per bit. It applies the bridge to physically implemented logically irreversible operations: erasure, reset, many-to-one overwrite, and irreversible compression. In this paper, Landauer enters as an update-throughput constraint on finite distinguishability, not as a replacement for the Bekenstein or holographic bounds.

Observer-dependence and finite records

Operational and relational views of physics emphasize that measurements are records available to physical systems rather than disembodied facts [25, 26]. Quantum Darwinism studies how information about a system becomes redundantly recorded in an environment [27]. Decoherence explains why stable records emerge in open quantum systems [27, 28]. These traditions motivate the idea that observerhood is a physical record-bearing role.

The FDS observer is not defined by consciousness or biological subjectivity. It is a finite distinction-register: a physical system capable of registering, preserving, updating, and ordering distinctions under finite capacity. That definition makes information bounds directly relevant to observation.

Rate-distortion and bounded representation

Rate-distortion theory asks how many bits are required to represent a source under a specified distortion level [29–32]. FDS uses this language to define capacity deficit: a task can demand more distinctions than a system can maintain. In physical settings, the available capacity is not only computational but also bounded by energy, causal access, and horizon area.

Relation to standard information thermodynamics

Each ingredient in FDS-T1—Bekenstein bounds, holographic ceilings, Landauer erasure, rate-distortion theory—is individually well established. The novelty is not any one ingredient taken separately. It is the finite-observer maintenance accounting obtained when task-relevant rate-distortion demand is compared against a bottlenecked accessible capacity (internal memory, boundary ceiling, channel, update rate, and thermodynamic budget) and then converted into compulsory exits or irreversible update cost. Bekenstein and holographic bounds set the storage or access ceiling; Landauer sets the per-irreversible-operation heat floor; rate-distortion sets the task demand. FDS-T1's contribution is the deficit-driven classification that connects them: when demand exceeds the bottlenecked accessible capacity, the system must coarse-grain, externalize, prune, relax the task, or fail; and the surplus distinctions that are irreversibly erased carry a Landauer-accounted thermodynamic cost.

FDS PRELIMINARIES

Distinctions and finite projections

Definition 1 (Distinction)**.** *A distinction is an operation or relation that separates at least two alternatives within a possibility space. At the elementary formal level, a distinction may be represented by a nontrivial partition*

$$D = \{A_i\}_{i=1}^{n}, \qquad A_i \cap A_j = \varnothing \ (i \neq j), \qquad \bigcup_i A_i = X, \tag{5}$$

with $n \geq 2$.

A physical distinction is not merely a formal partition. It must be instantiated by a record, boundary, coupling, memory state, detector state, or other physical carrier. A finite system implements distinctions through a finite projection.

Definition 2 (Finite distinction projection)**.** *Let* Ω *be a physical possibility space for a region or task. A finite observer* $\mathcal{O}$ *implements a finite distinction projection*

$$\pi_{\mathcal{O}} : \Omega \to \mathcal{Z}_{\mathcal{O}}, \tag{6}$$

where $\mathcal{Z}_{\mathcal{O}}$ *is the finite set of distinction classes accessible to* $\mathcal{O}$.

Definition 3 (Distinguishability budget)**.** *The instantaneous distinguishability budget of* $\mathcal{O}$ *is*

$$N_{\mathcal{O}} = |\mathcal{Z}_{\mathcal{O}}| = |\mathrm{Im}(\pi_{\mathcal{O}})|. \tag{7}$$

The bit capacity is

$$C_{\mathcal{O}} = \log_2 N_{\mathcal{O}}. \tag{8}$$

When $\mathcal{Z}_{\mathcal{O}}$ *is continuous or effectively continuous,* $C_{\mathcal{O}}$ *denotes the operational number of reliably distinguishable states under the observer's noise, energy, resolution, and error-tolerance constraints.*

Record-bearing observers

Definition 4 (Finite distinction-register)**.** *A finite distinction-register is a physical system that can register, preserve, update, and order distinctions using finite resources. It must have a record carrier, a boundary or interface, a stability condition, and a finite update capacity.*

This definition intentionally excludes two extremes. A disembodied mathematical observer is not a finite distinction-register because it has no physical record carrier. A purely passive boundary that never records or updates distinctions is also not an observer in this sense. A detector, memory device, organism, autonomous system, or laboratory apparatus can be a finite distinction-register when its boundary and records satisfy the relevant stability and update conditions.

Stock capacity and update throughput

The word "capacity" hides two distinct quantities: storage-like capacity and flow-like update throughput.

Definition 5 (Stock / accessible-state capacity)**.** *The stock capacity of observer* $\mathcal{O}$ *in region* $\mathcal{D}$ *at time* t *is*

$$\begin{aligned} C_{\mathrm{stock}}(\mathcal{O}, \mathcal{D}, t) = \min\{&C_{\mathrm{mem}}(\mathcal{O}, t),\, C_{\mathrm{bd}}(\mathcal{O}, \mathcal{D}, t), \\ &C_{\mathrm{chan}}(\mathcal{O}, \mathcal{D}, t),\, C_{\mathrm{ext}}^{\mathrm{eff}}(\mathcal{O}, t)\}, \end{aligned} \tag{9}$$

where C_{mem} *is internal memory capacity,* C_{bd} *is boundary-limited capacity,* C_{chan} *is channel-limited capacity, and* $C_{\mathrm{ext}}^{\mathrm{eff}}$ *is effective externalized capacity after indexing, verification, synchronization, latency, and repair costs.*

Definition 6 (Update / irreversible-throughput capacity)**.** *The maximum number of irreversible distinction updates over interval* $[t, t+\tau]$ *is bounded by the available free energy or allowable dissipation budget:*

$$I_{\mathrm{update}}^{\max}(t, t+\tau) \leq \int_t^{t+\tau} \frac{\dot{F}_{\mathrm{avail}}(s)}{k_{\mathrm{B}} T(s) \ln 2}\, ds, \tag{10}$$

where $\dot{F}_{\mathrm{avail}}$ *is the rate of available free-energy input or allowable dissipation for irreversible operations. This is not a holographic bound. It is an update-throughput ceiling derived from the Landauer bridge: the actual heat cost of irreversible updates satisfies* $\dot{Q}_{\mathrm{info}}(t) \geq k_{\mathrm{B}} T(t) \ln 2\, \dot{I}_{\mathrm{irr}}(t)$, *so the available budget* $\dot{F}_{\mathrm{avail}}$ *upper-bounds the rate at which irreversible updates can be physically sustained.*

The effective task-available capacity is the minimum of stock and throughput:

$$C_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}, t, \tau) = \min\{\, C_{\mathrm{stock}}(\mathcal{O}, \mathcal{D}, t),\ I_{\mathrm{update}}^{\max}(t, t+\tau)\,\}. \tag{11}$$

Unless otherwise stated, all capacity terms entering C_{acc} are converted into usable task-relevant bits over the same update window $[t, t+\tau]$. Stock capacities are interpreted as the maximum number of simultaneously maintained distinctions that can be reliably preserved during the window, while throughput capacities are interpreted as the maximum number of irreversible updates that can be executed during the window. This conversion ensures that the minimum in (11) compares quantities with compatible units. When no ambiguity arises, we write $C_{\mathrm{acc}}(\mathcal{O}, \mathcal{D})$ as shorthand for $C_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}, t, \tau)$.

Definition 7 (Boundary-relative capacity deficit)**.**

$$\Delta_{\mathrm{FDS}}(\mathcal{O}, \mathcal{D}, \Psi, \varepsilon, \tau) = R^{(\tau)}_{\mathrm{min}}(\varepsilon; \Psi) - C_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}, t, \tau). \tag{12}$$

PHYSICAL BRIDGE ASSUMPTIONS

Finite distinguishability

Assumption 1 (Finite distinguishability)**.** *A physically instantiated system that maintains identity has a finite distinguishability budget over any finite operational interval. Its accessible records, memory states, update channels, and boundary-maintenance operations are bounded by finite physical resources.*

This assumption is deliberately weak. It does not require that all physical systems saturate a black-hole bound. It requires only that physical systems do not possess infinite usable distinguishability. It is compatible with finite entropy bounds, finite channel capacities, finite memory, finite energy, finite precision, and finite causal access.

Bekenstein bridge

Assumption 2 (Bekenstein bridge)**.** *For a weakly gravitating physical system of total energy E confined to radius R, the entropy and information accessible within that region obey the Bekenstein bound*

$$S \leq \frac{2\pi k_{\mathrm{B}} E R}{\hbar c}, \qquad I \leq \frac{2\pi E R}{\hbar c \ln 2}. \tag{13}$$

FDS does not derive this bound in the present paper. It uses it as an established physical bridge constraint.

Holographic bridge

Assumption 3 (Holographic ceiling)**.** *For a horizon-bounded or gravitationally saturated region whose appropriate boundary area is A, the maximum accessible entropy satisfies*

$$S \leq \frac{k_{\mathrm{B}} A}{4\ell_{\mathrm{P}}^2}, \tag{14}$$

and therefore the maximum number of accessible bits satisfies

$$I \leq \frac{A}{4\ell_{\mathrm{P}}^2 \ln 2}. \tag{15}$$

The qualifier "appropriate boundary" matters. In covariant settings the relevant area may be defined by a light-sheet or causal horizon rather than an arbitrary spatial surface [14].

Landauer update bridge

Assumption 4 (Landauer update bridge)**.** *When a physical system performs a logically irreversible operation—erasure, many-to-one overwrite, reset, irreversible compression, or pruning of previously distinguishable alternatives—the operation incurs a thermodynamic entropy cost under standard Landauer conditions.*

This bridge is used only for update and erasure accounting. Stable storage and reversible evolution are not automatically charged with $k_{\mathrm{B}} T \ln 2$ per bit.

Finite causal access

Assumption 5 (Finite causal access)**.** *A finite physical observer cannot instantaneously access, register, update, and causally use arbitrary distant distinctions without unbounded update capacity. Distinguishability is therefore restricted by causal structure and finite update rate.*

This assumption is weaker than a derivation of relativity. It says only that finite observers require finite causal access. The numerical value of c is not derived here.

FINITE DISTINGUISHABILITY BUDGETS

The finite-budget proposition

Proposition 1 (Finite record carriers imply finite distinguishability)**.** *Let $\mathcal{O}$ be a physical record-bearing system with finite internal record capacity, finite error tolerance, and finite update resources over interval $[t_0, t_1]$. Then $\mathcal{O}$ has a finite operational distinguishability budget over that interval.*

Proof. A distinction available to $\mathcal{O}$ must be encoded in a record or state difference that can be reliably discriminated under the observer's noise and error tolerance. If the record substrate has finite capacity, only finitely many such distinguishable record states can be reliably preserved and accessed over the interval. If update resources are also finite, the number of new distinctions that can be acquired, overwritten, corrected, or stabilized is finite. Therefore the operational image of $\pi_{\mathcal{O}}$ is finite, and $N_{\mathcal{O}} < \infty$. □

Remark 1. *This proposition is not a statement that the underlying mathematical Hilbert space is finite-*

dimensional. It is a statement about operationally accessible distinctions for a finite physical record carrier.

Distinguishability throughput

A finite observer may have both an instantaneous storage budget and a lifetime throughput budget. Let $\dot{I}_{\rm irr}(t)$ be the rate at which logically irreversible distinction updates occur. Under the Landauer bridge,

$$\dot{Q}_{\rm info}(t) \geq k_{\rm B}T(t)\ln 2\; \dot{I}_{\rm irr}(t). \tag{16}$$

If the observer has finite free-energy input or dissipation allowance over $[t_0, t_1]$, then

$$\int_{t_0}^{t_1} \dot{I}_{\rm irr}(t)\, dt \leq \int_{t_0}^{t_1} \frac{\dot{Q}_{\rm info}(t)}{k_{\rm B}T(t)\ln 2}\, dt, \tag{17}$$

with the inequality interpreted carefully when temperature varies or non-thermal reservoirs are involved. This is not a holographic bound. It is an update-throughput bound. It says that a finite observer cannot indefinitely acquire and erase distinctions for free.

Internal versus accessible budgets

Internal memory can be far below the physical entropy ceiling of the region. Conversely, a large physical region can contain many degrees of freedom that are inaccessible to a small observer. FDS therefore uses the bottleneck form (11). For observation and control tasks, what matters is not the total number of microstates in a region but the number of task-relevant distinctions that can be reliably acquired, represented, and used by the observer.

Theorem 1 (Accessible budget bottleneck)**.** *For any finite observer $\mathcal{O}$ performing a task in region $\mathcal{D}$ over window τ, the operational capacity available to that task is upper-bounded by the minimum of stock capacity and update throughput:*

$$C_{\rm task}(\mathcal{O}, \mathcal{D}, t, \tau) \leq C_{\rm acc}(\mathcal{O}, \mathcal{D}, t, \tau). \tag{18}$$

Proof. A task-relevant distinction must pass multiple filters: it must be storable in memory or externalized records, physically available through the boundary, transmissible through a finite channel, and updatable within the relevant timescale. The effective usable capacity is the minimum of the stock bottleneck (memory, boundary, channel, externalization) and the update-throughput bottleneck. Therefore $C_{\rm task}$ cannot exceed $C_{\rm acc}$. □

HOLOGRAPHIC INFORMATION BOUNDS AS DISTINGUISHABILITY CEILINGS

Weak-gravity regime: Bekenstein ceiling

In weakly gravitating bounded systems, the Bekenstein bound gives

$$C_{\rm bd}(\mathcal{O}, \mathcal{D}) \leq \frac{2\pi ER}{\hbar c \ln 2}, \tag{19}$$

where E and R characterize the bounded region relative to the operational setup.

Theorem 2 (Bekenstein distinguishability ceiling)**.** *Assume the Bekenstein bridge applies to a bounded physical region $\mathcal{D}$ of energy E and radius R. Then no finite observer can operationally distinguish more than*

$$N_{\mathcal{O}} \leq \exp\left(\frac{2\pi ER}{\hbar c}\right) \tag{20}$$

region-states as mutually accessible alternatives through that region without violating the entropy bound.

Proof. The Bekenstein bridge bounds the entropy by $S/k_{\rm B} \leq 2\pi ER/\hbar c$. The number of equiprobable distinguishable alternatives compatible with entropy S is $N = \exp(S/k_{\rm B})$. Therefore $N \leq \exp(2\pi ER/\hbar c)$. In bits this is equivalent to (19). □

Remark 2. *This theorem is conditional on the physical bridge. It does not derive the Bekenstein bound from the distinction primitive alone.*

Remark 3. *Here $N_{\mathcal{O}}$ should be read as the maximum effective number of reliably distinguishable, thermodynamically admissible alternatives, or equivalently the equiprobable support size associated with the entropy ceiling. For nonuniform ensembles, the corresponding quantity is the effective entropy number rather than a raw set cardinality.*

Horizon-saturating regime: area ceiling

When gravitational collapse or horizon saturation becomes relevant, the area law dominates the naive volume count.

Theorem 3 (Holographic distinguishability ceiling)**.** *Assume the holographic ceiling applies to a causal or horizon-bounded region $\mathcal{D}$ with boundary area A. Then the accessible distinction capacity satisfies*

$$C_{\mathcal{O}} \leq \frac{A}{4\ell_{\rm P}^2 \ln 2}. \tag{21}$$

Equivalently,

$$N_{\mathcal{O}} \leq \exp\left(\frac{A}{4\ell_{\rm P}^2}\right). \tag{22}$$

Proof. The holographic bridge gives $S \leq k_{\mathrm{B}} A/(4\ell_{\mathrm{P}}^2)$. The number of distinguishable alternatives is at most $\exp(S/k_{\mathrm{B}})$, hence $N_{\mathcal{O}} \leq \exp(A/4\ell_{\mathrm{P}}^2)$. Taking $\log_2$ yields (21). □

Why area, not volume

The FDS interpretation of area scaling is operational. A finite observer interacts with a region through a boundary or causal interface. Increasing interior volume without increasing boundary capacity cannot indefinitely increase the number of distinctions that can be accessed, stabilized, and communicated across that boundary. At gravitational saturation, attempts to store or distinguish more interior alternatives change the boundary conditions themselves, producing horizon behavior rather than unlimited volume scaling.

This is not a claim that ordinary low-energy systems always saturate area bounds. Most do not. The claim is that the ultimate ceiling on accessible distinguishability is boundary-like in the regimes where gravity and causal horizons enforce the strongest known entropy constraints.

Comparison of ceilings

Combining the internal, Bekenstein, and holographic limits gives a conservative stock-capacity formula. The boundary-limited component of stock capacity is

$$C_{\mathrm{bd}}(\mathcal{O},\mathcal{D}) \leq \min\left\{\frac{2\pi ER}{\hbar c \ln 2},\ \frac{A}{4\ell_{\mathrm{P}}^2 \ln 2}\right\}. \tag{23}$$

The full stock capacity is

$$C_{\mathrm{stock}}(\mathcal{O},\mathcal{D},t) = \min\{C_{\mathrm{mem}}(\mathcal{O},t),\ C_{\mathrm{bd}}(\mathcal{O},\mathcal{D},t),\ C_{\mathrm{chan}}(\mathcal{O},\mathcal{D},t),\ C^{\mathrm{eff}}_{\mathrm{ext}}(\mathcal{O},t)\}, \tag{24}$$

and the effective task capacity is

$$C_{\mathrm{acc}}(\mathcal{O},\mathcal{D},t,\tau) = \min\{\, C_{\mathrm{stock}}(\mathcal{O},\mathcal{D},t),\ I^{\max}_{\mathrm{update}}(t,t+\tau)\,\}. \tag{25}$$

Different terms dominate in different regimes. For a laboratory detector, C_{mem}, C_{chan}, or $C^{\mathrm{eff}}_{\mathrm{ext}}$ may dominate. For black holes or cosmological horizons, the area term dominates. For thermally constrained computational systems, the update-throughput term dominates.

LOCAL OBSERVERS AND CAUSAL PATCHES

Observer-relative patches

A finite observer does not resolve the universe globally. It resolves a causal patch.

Definition 8 (Observer patch). *The observer patch $\mathcal{D}_{\mathcal{O}}(t)$ is the region whose states can in principle influence the records of $\mathcal{O}$ by time t under the applicable causal and channel constraints.*

The boundary $\partial\mathcal{D}_{\mathcal{O}}$ may be a literal detector surface, a light cone, a causal diamond boundary, a black-hole horizon, an apparent horizon, an acoustic horizon in analogue systems, or a task-defined operational boundary.

Definition 9 (Distinguishability horizon). *A distinguishability horizon for $\mathcal{O}$ is a boundary beyond which differences in physical state cannot be operationally registered, preserved, and used by $\mathcal{O}$ within the relevant resource and timescale constraints.*

A distinguishability horizon need not be an event horizon. It may arise from noise, finite sampling, finite memory, finite control, finite latency, or thermodynamic cost. Event horizons and causal horizons are limiting physical cases.

Patch nesting

Remark 4 (Nested patches). *If $\mathcal{D}_1 \subseteq \mathcal{D}_2$ are observer-accessible regions with compatible boundary definitions, the accessible boundary capacity of $\mathcal{D}_1$ cannot exceed that of $\mathcal{D}_2$ under otherwise identical conditions. Operational capacity, however, is a minimum over multiple bottlenecks; a smaller patch may have better channel or update characteristics.*

Composite observers

Composite observers can pool records, distribute memory, and externalize distinctions into shared media. But pooling does not remove physical bounds. It changes the boundary.

Definition 10 (Composite finite observer). *A composite finite observer is a network of finite distinction-registers coupled by communication channels, shared records, or coordination protocols, treated as a higher-level observer only when the network can preserve and update joint distinctions with sufficient reliability.*

Proposition 2 (Externalization changes the budget boundary). *When an observer externalizes records into an environment, database, instrument, institution, or horizon boundary, the relevant budget becomes the budget of the coupled observer–record system, not the internal memory alone.*

Proof. Externalized records are useful only if the observer can access, validate, update, and protect them through a boundary. The operational distinction is therefore carried by the coupled system. The capacity is no longer merely the internal memory capacity $C_{\mathrm{mem}}(\mathcal{O}, t)$ but is bounded by the record medium, access channel, stability condition, and the boundary information ceiling of the enlarged system. □

CAPACITY DEFICIT UNDER HOLOGRAPHIC BOUNDS

Physical distinction demand

For a task family Ψ, the demand $R^{(\tau)}_{\mathrm{min}}(\varepsilon; \Psi)$ may include distinctions among microstates, macrostates, trajectories, causal histories, control-relevant variables, or boundary-relevant signals. The FDS-T1 deficit is positive when

$$R^{(\tau)}_{\mathrm{min}}(\varepsilon; \Psi) > C_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}, t, \tau). \tag{26}$$

A positive deficit is not ignorance in the ordinary sense. It is an operational impossibility under the chosen boundary, resources, and timescale.

Budget-exit theorem

Theorem 4 (Budget-exit theorem)**.** *Let $\mathcal{O}$ be a finite distinction-register performing task family Ψ over region $\mathcal{D}$ with distortion tolerance ε and update window τ. Under fixed observer boundary, resource budget, task family, and update window, if $\Delta_{\mathrm{FDS}}(\mathcal{O}, \mathcal{D}, \Psi, \varepsilon, \tau) > 0$ persists over the relevant interval, then $\mathcal{O}$ cannot maintain full-fidelity task performance using internal accessible capacity alone. It must enter at least one exit class: coarse-graining or compression, externalization, pruning, task relaxation, or failure. Increasing the budget or changing the observer boundary constitutes a change of system, not a violation of the theorem.*

Proof. Full-fidelity task performance requires at least $R^{(\tau)}_{\mathrm{min}}(\varepsilon; \Psi)$ bits of relevant distinction capacity. By assumption this exceeds C_{acc}, which is the minimum of stock capacity and update throughput. Under fixed boundary, resources, and task window, the missing distinctions cannot be generated internally without violating the capacity ceiling. The admissible exits are to reduce demand through coarse-graining or compression, externalize part of the burden, prune or discard distinctions, relax the task, or fail to perform it. □

Holographic deficit

In a horizon-bounded regime, the deficit uses the boundary-limited stock component:

$$\Delta^{\mathrm{hol}}_{\varepsilon}(\mathcal{O}, \tau) = R^{(\tau)}_{\mathrm{min}}(\varepsilon; \Psi) - \min\left\{ \begin{array}{l} C_{\mathrm{stock}}(\mathcal{O}, \mathcal{D}, t), \\ I^{\mathrm{max}}_{\mathrm{update}}(t, t+\tau) \end{array} \right\}. \tag{27}$$

When $\Delta^{\mathrm{hol}}_{\varepsilon} > 0$, no increase in volume description alone solves the problem. The limiting factor is boundary-accessible distinction capacity.

Observer-relative entropy

Definition 11 (Observer-relative accessible entropy)**.** *The accessible entropy assigned by observer $\mathcal{O}$ to region $\mathcal{D}$ is*

$$S_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}) = k_{\mathrm{B}} \ln N_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}), \tag{28}$$

where N_{acc} is the number of region alternatives operationally distinguishable by $\mathcal{O}$ under its boundary, channel, record, and update constraints.

Corollary 1 (Entropy bottleneck)**.** *The accessible entropy satisfies*

$$S_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}) \le k_{\mathrm{B}} \ln 2 \, C_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}). \tag{29}$$

In a holographic regime,

$$S_{\mathrm{acc}}(\mathcal{O}, \mathcal{D}) \le \frac{k_{\mathrm{B}} A}{4\ell_{\mathrm{P}}^2}. \tag{30}$$

This formulation permits observer-relative entropy without making entropy subjective. The bound is physical; the accessible coarse-graining depends on the finite observer.

MAINTENANCE INEQUALITY

Distinction deficit and irreversible update

Positive capacity deficit forces the observer to manage surplus distinctions that cannot be stored or transmitted within the accessible budget. Let

$$[\Delta_{\mathrm{FDS}}]_+ = \max(\Delta_{\mathrm{FDS}}, 0), \tag{31}$$

let C_{inv} be invariant-supported compression credit, and let $C^{\mathrm{gain}}_{\mathrm{ext}}$ be net externalization gain. The surplus distinctions that must be irreversibly handled are

$$I_{\mathrm{irr}}(t, \tau) \ge \left[\Delta_{\mathrm{FDS}}(t, \tau) - C_{\mathrm{inv}}(t, \tau) - C^{\mathrm{gain}}_{\mathrm{ext}}(t, \tau) \right]_+ . \tag{32}$$

Under the Landauer bridge, each logically irreversible operation has a thermodynamic lower bound:

$$\dot{Q}_{\mathrm{maint}}(t) \ge k_{\mathrm{B}} T(t) \ln 2 \, \frac{I_{\mathrm{irr}}(t, \tau)}{\tau} + \dot{Q}_{\mathrm{phys}}(t), \tag{33}$$

where $\dot{Q}_{\mathrm{phys}}$ includes control, coupling, isolation, leakage, transport, error correction, and measurement overhead.

Remark 5. *The Landauer term applies only to logically irreversible operations. Stable storage, read-only access, reversible computation, and unitary evolution are not automatically charged* $k_{\rm B}T \ln 2$ *per bit.*

Minimal no-rescue corollary

If $C_{\rm inv} = 0$, $C_{\rm ext}^{\rm gain} = 0$, and $\dot{Q}_{\rm phys} = 0$, then

$$\dot{Q}_{\rm maint}(t) \geq k_{\rm B}T(t) \ln 2 \, \frac{[\Delta_{\rm FDS}(t,\tau)]_+}{\tau}. \tag{34}$$

Interpretation. In the absence of invariant compression, externalization gain, and compensating physical channels, positive distinction deficit directly sets a Landauer-style lower bound on irreversible maintenance cost. This is a minimal lower-bound regime, not a universal full-cost formula.

BUDGET CROSSING AS TESTABLE PREDICTION

Crossing parameter

Define the instantaneous crossing parameter

$$\chi(t) = R_{\rm min}^{(\tau)}(\varepsilon; \Psi) - C_{\rm acc}(\mathcal{O}, \mathcal{D}, t, \tau).$$

As $\chi(t)$ crosses zero from negative to positive, the observer moves from budget surplus to budget deficit.

Expected observables

When $\chi > 0$ persists, finite observers should exhibit:

- increased task error,
- increased latency,
- increased compression ratio,
- coarser partitions,
- increased external memory use,
- increased reset / garbage collection events,
- increased heat output under irreversible update,
- pruning of maintained states,
- task relaxation,
- failure.

Test classes

Test class 1: Bounded-memory computation. Build or simulate a bounded-memory reversible computation system. Feed continuous task streams. Vary memory budget. Track garbage accumulation, reset frequency, error correction, compression, heat, and task failure. Expected result: reversible gates may avoid local erasure cost, but open-ended bounded-memory operation eventually requires housekeeping, compression, reset, pruning, or entropy export.

Test class 2: Finite sensor/detector system. Define observer boundary, estimate internal memory and channel capacity, define task family and distortion tolerance, estimate $R_{\rm min}$. Increase task complexity until $R_{\rm min}$ crosses $C_{\rm acc}$. Observe error, latency, compression, or externalization.

Test class 3: Analogue-horizon system. Use an analogue horizon or engineered causal-access bottleneck. Measure recoverable distinguishable modes. Look for degradation as cross-boundary access becomes unavailable.

A MINIMAL COMPUTABLE MODEL OF BUDGET CROSSING

The maintenance bound is abstract. To make it computable, we construct a minimal model of an observer crossing a causal-access bottleneck.

Model setup

Let there be an external process $Z_t \in \mathcal{Z}$. A finite observer observes Z_t through a restricted channel Y_t and maintains an internal memory state M_t that carries task-relevant distinctions. The stock capacity is

$$C_{\rm stock}(t) = \min\left\{C_M,\ C_Y(t),\ C_H(t),\ C_{\rm ext}^{\rm eff}(t)\right\}, \quad (35)$$

where C_M is fixed memory capacity, $C_Y(t)$ is the channel capacity at time t, $C_H(t)$ is the causal-boundary capacity, and $C_{\rm ext}^{\rm eff}$ is effective externalization capacity. The full accessible capacity including update-throughput bound is

$$C_{\rm acc}(t) = \min\left\{C_{\rm stock}(t),\ I_{\rm update}^{\max}(t, t+\tau)\right\}, \quad (36)$$

where $I_{\rm update}^{\max}$ follows Eq. (10).

Model the causal boundary crossing as a sigmoid transition:

$$C_H(t) = C_0\,\eta(t), \qquad \eta(t) = \frac{1}{1+\exp((t-t_h)/\sigma)}, \quad (37)$$

where t_h is the crossing time and σ is the transition width. This does not claim to model a real black hole; it is a tunable causal-access bottleneck that captures the qualitative behavior of a horizon-like collapse.

Observer-relative entropy divergence

This subsection is a finite-budget diagnostic inside the toy model. Quantum-foundational applications to Wigner-type scenarios or measurement interpretations are outside the scope of FDS-T1.

Define the observer's residual uncertainty about the external state:

$$S_{\mathcal{O}}(t) = H(Z_t | M_t, Y_{\le t}^{\rm acc}). \quad (38)$$

As $C_{\rm acc}(t)$ decreases, M_t can carry fewer task-relevant distinctions, so $S_{\mathcal{O}}(t)$ increases. Two observers $\mathcal{O}_1, \mathcal{O}_2$ with different $C_{\rm acc}^{(1)}(t), C_{\rm acc}^{(2)}(t)$ will assign different entropies:

$$\Delta S_{\mathcal{O}_1,\mathcal{O}_2}(t) = S_{\mathcal{O}_1}(t) - S_{\mathcal{O}_2}(t). \quad (39)$$

This divergence is directly computable from the model parameters.

Coarse-graining error floor

The most direct link to testable prediction is through rate-distortion theory. Let $D_{\min}(R)$ be the minimal achievable distortion for process Z_t at rate R. The finite observer's error floor is

$$\varepsilon_{\min}(t) = D_{\min}(C_{\rm acc}(t)). \quad (40)$$

The task is maintainable if and only if $D_{\min}(C_{\rm acc}(t)) \le \varepsilon$. Define the collapse time as

$$t_c = \inf\left\{t :\ D_{\min}(C_{\rm acc}(t)) > \varepsilon\right\}. \quad (41)$$

Gaussian source example

If Z_t is a Gaussian source with variance σ_Z^2 and the distortion is mean-squared error, the rate-distortion function is

$$R(D) = \frac{1}{2}\log_2\frac{\sigma_Z^2}{D}, \qquad D_{\min}(R) = \sigma_Z^2\,2^{-2R}. \quad (42)$$

The error floor becomes

$$\boxed{\varepsilon_{\min}(t) = \sigma_Z^2\,2^{-2C_{\rm acc}(t)}}. \quad (43)$$

When the causal-access bottleneck drives $C_{\rm acc}(t) \to 0$, the error floor rises to σ_Z^2—the observer degrades to guessing the prior variance.

Binary source example

If $Z_t \in \{0,1\}$ with $P(Z=1) = p$ and Hamming distortion, the rate-distortion function in the range $0 \le D \le \min(p, 1-p)$ is

$$R(D) = H_b(p) - H_b(D), \quad (44)$$

where H_b is the binary entropy function. The error floor is

$$D_{\min}(t) = H_b^{-1}\big(H_b(p) - C_{\rm acc}(t)\big). \quad (45)$$

As $C_{\rm acc} \to 0$, $D_{\min} \to \min(p, 1-p)$: the observer can only guess the most probable state.

Bottleneck-switching kink theorem

The error-floor predictions above can be generalized into an analytic theorem. Let the accessible capacity be a pointwise minimum of differentiable capacity channels:

$$C_{\rm acc}(t) = \min_i C_i(t). \tag{46}$$

Suppose that at $t = t^*$ the active bottleneck switches from C_j to C_k, with

$$C_j(t^*) = C_k(t^*), \qquad \dot{C}_j(t^*) \neq \dot{C}_k(t^*), \tag{47}$$

and all other $C_i(t^*)$ are strictly larger. Let the distortion floor be

$$\varepsilon_{\min}(t) = D_{\min}(C_{\rm acc}(t)), \tag{48}$$

where $D_{\min}(R)$ is differentiable and $D'_{\min}(C_{\rm acc}(t^*)) \neq 0$.

Theorem 5 (Bottleneck-switching kink)**.** *Under conditions (46)–(48), the error floor $\varepsilon_{\min}(t)$ generically has a slope discontinuity (kink) at the switch time t^*.*

Proof. For $t < t^*$ in a neighborhood of the switch, $C_{\rm acc}(t) = C_j(t)$, while for $t > t^*$, $C_{\rm acc}(t) = C_k(t)$. Therefore

$$\left.\frac{d\varepsilon_{\min}}{dt}\right|_{t^*_-} = D'_{\min}(C_{\rm acc}(t^*))\,\dot{C}_j(t^*), \tag{49}$$

$$\left.\frac{d\varepsilon_{\min}}{dt}\right|_{t^*_+} = D'_{\min}(C_{\rm acc}(t^*))\,\dot{C}_k(t^*). \tag{50}$$

Since $D'_{\min}(C_{\rm acc}(t^*)) \neq 0$ and $\dot{C}_j(t^*) \neq \dot{C}_k(t^*)$, the two one-sided derivatives differ. Hence the error floor exhibits a slope discontinuity—a kink—at the bottleneck switch. □

The theorem implies that *any* finite observer whose accessible capacity is a pointwise minimum of differentiable channel terms will generically produce observable kinks in its error floor at bottleneck transitions, regardless of the specific source or distortion measure, provided the rate-distortion function is differentiable at the operating point.

Memory update and erased distinctions

To connect to the Landauer bound, define the memory dynamics as

$$M_{t+\tau} = \Pi_{C_{\rm acc}(t+\tau)}\big[U(M_t, Y_t)\big], \tag{51}$$

where Π_C is a projection or compression operator that projects onto the C-bit accessible space. The number of erased distinctions per update step is

$$E_t = H\big(U(M_t, Y_t)\big) - H(M_{t+\tau}), \tag{52}$$

or more precisely, the logical erasure [3]:

$$E_t = H(M_t | M_{t+\tau}, Y_t). \tag{53}$$

The Landauer bridge then gives $\dot{Q}_{\rm irr}(t) \geq k_B T \ln 2 \cdot E_t/\tau$, linking the toy model directly to the maintenance bound (33) in the Maintenance inequality section.

Numerical demonstration and predictions

The numerical parameters in this section are illustrative and are chosen to demonstrate the analytic bottleneck mechanism, not to fit a specific physical black-hole or analogue-horizon platform. The Bottleneck-Switching Kink Theorem (Theorem 5) is confirmed numerically by a minimal finite-observer model with four capacity channels (fixed memory, decreasing channel, decreasing causal-boundary via sigmoid, decreasing thermodynamic budget) driven by a Gaussian source under mean-squared error distortion and a binary source under Hamming distortion. The model uses the minimal no-rescue regime defined above and is fully reproducible (Code Availability).

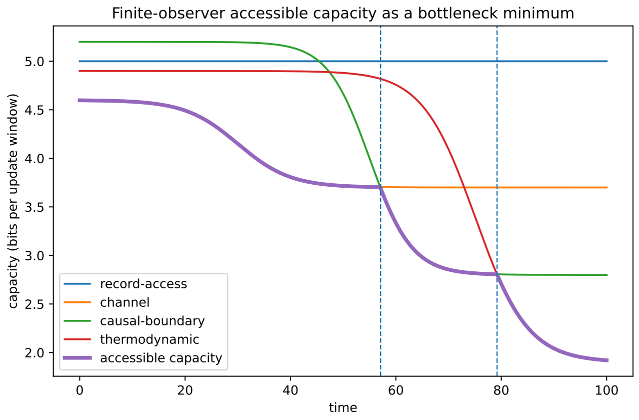

FIG. 1. Capacity bottleneck switching. Four capacity channels and their pointwise minimum $C_{\mathrm{acc}}(t)$. Vertical dashed lines mark bottleneck transitions.

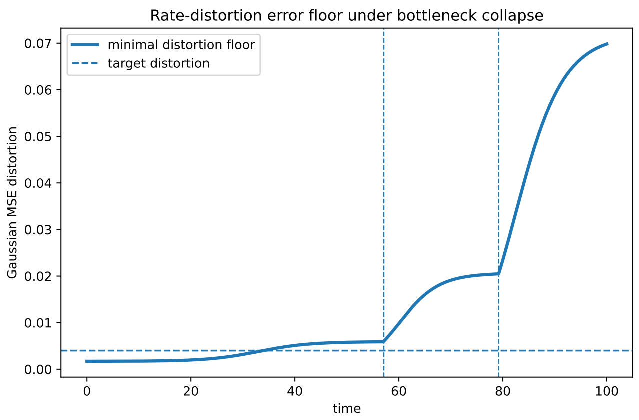

FIG. 2. Gaussian error-floor kinks. The rate-distortion error floor $\varepsilon_{\min}(t) = \sigma_Z^2\, 2^{-2C_{\mathrm{acc}}(t)}$ shows clear slope discontinuities (kinks) at each bottleneck switch (vertical lines). Gray dashed lines mark the one-sided derivative extrapolations.

Prediction 1 (capacity collapse curve). As capacity channels degrade, $C_{\mathrm{acc}}(t) = \min_i C_i(t)$ switches between bottleneck regimes (Fig. 1). Each switch produces a kink in $C_{\mathrm{acc}}(t)$ and therefore a slope change in the error floor $\varepsilon_{\min}(t) = D_{\min}(C_{\mathrm{acc}}(t))$ (Fig. 2, Theorem 5). A binary source produces analogous kinks under Hamming distortion.

Prediction 2 (observer entropy divergence). Two observers with different $C^{(1)}_{\mathrm{acc}}(t), C^{(2)}_{\mathrm{acc}}(t)$ develop an entropy gap $\Delta S_{\mathcal{O}_1,\mathcal{O}_2}(t)$ that grows and then contracts as the later observer's bottleneck crosses (Fig. 4). The gap is a direct measure of observer-relative causal access.

Prediction 3 (erasure heat spike). If the system attempts to maintain the same distortion ε as $C_{\mathrm{acc}}(t)$ declines, $\Delta_{\mathrm{FDS}}(t)$ rises and the minimal no-rescue bound (34) produces a positive heat lower bound. Fig. 3 shows a transient heat spike during the causal-

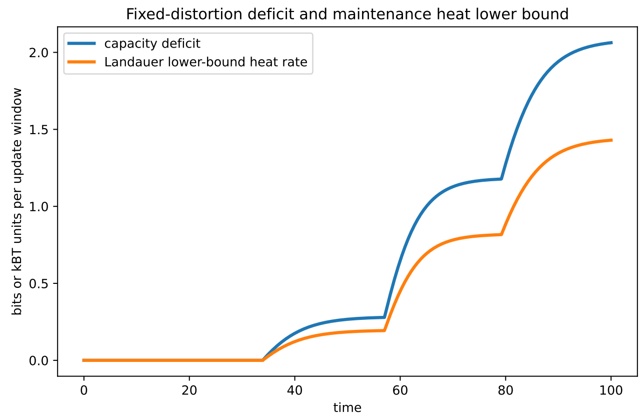

FIG. 3. Fixed-distortion capacity deficit and Landauer heat lower bound. The capacity deficit $\Delta_{\mathrm{FDS}}(t)$ (bits) and its corresponding minimal no-rescue heat lower bound $\dot{Q}_{\mathrm{maint}}(t)/(k_B T \ln 2)$ from (34) rise as accessible capacity falls below task demand.

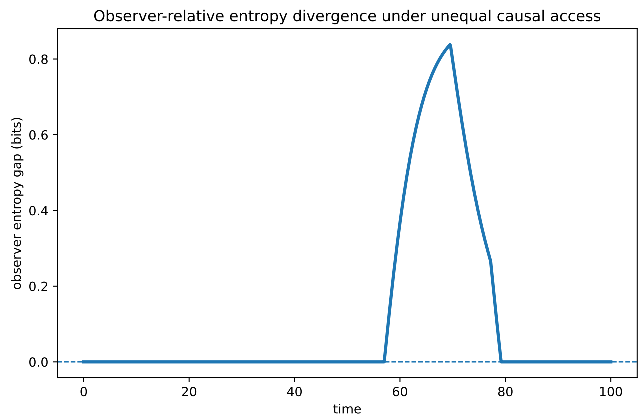

FIG. 4. Observer-relative entropy divergence. Two observers with staggered causal-boundary transitions ($t_h^{(1)} = 55$, $t_h^{(2)} = 67$) develop a transient entropy gap $\Delta S_{\mathcal{O}_1,\mathcal{O}_2}(t)$ that peaks during the interval between their respective bottleneck crossings.

boundary collapse phase. The height of the spike is proportional to the peak deficit and is bounded below by $k_B T \ln 2 \cdot \Delta_{\mathrm{FDS}}/\tau$.

SIMULATION AND EXPERIMENTAL PROTOCOLS

We describe three tiers of protocol for testing the FDS maintenance bound, ordered by increasing physical realism and difficulty.

Protocol A: Constrained-memory channel simulation

Goal. Verify $\varepsilon_{\min}(t) = D(C_{\rm acc}(t))$ and $\dot{Q}_{\rm irr} \geq k_B T \ln 2 \sum_t H(M_t|M_{t+\tau}, Y_t)$ in a controlled simulation.

System. Construct a source Z_t (e.g., Gaussian AR(1) process, binary Markov chain, hidden Markov model, or chaotic logistic map). The observer has:

- fixed memory C_M (bits);
- channel capacity $C_Y(t)$;
- causal-boundary capacity $C_H(t) = C_0\,\eta(t)$ with $\eta(t)$ given by (37);

- stock capacity $C_{\rm stock}(t) = \min\{C_M, C_Y(t), C_H(t), C^{\rm eff}_{\rm ext}\}$, and accessible capacity $C_{\rm acc}(t) = \min\{C_{\rm stock}(t), I^{\max}_{\rm update}(t, t+\tau)\}$, evaluated per update window;

Procedure.

1. Pre-register a task distortion tolerance ε.
2. The observer tracks Z_t using an optimal or bounded encoder.
3. Gradually reduce $C_H(t)$ or $C_Y(t)$.
4. Record reconstruction error, retained mutual information $I(Z_t; M_t)$, erased bits $H(M_t|M_{t+\tau}, Y_t)$, reset or compression frequency, and estimated heat lower bound.

Expected result. When $R^{(\tau)}_{\min}(\varepsilon) > C_{\rm acc}(t)$, at least one of the following must occur: error rises, erasure or compression rises, or externalization or pruning is triggered. If none occur while task performance is maintained, the FDS maintenance bound is violated.

Protocol B: Finite-temperature quantum memory and channel

System. A finite quantum memory M coupled to a bath at temperature T, receiving states from a quantum or classical source.

Controllable parameters. Memory Hilbert dimension d_M, channel transmissivity $\eta(t)$, reset frequency, bath temperature, task discrimination threshold, measurement or coarse-graining map.

Measurable quantities. Discrimination success probability, accessible mutual information, entropy production, heat dissipation, reset work, quantum memory fidelity, task-relevant distortion.

Core test. Tune $\eta(t) : 1 \to 0$, define $C_{\rm chan}(t) = \log_2 d_{\rm eff}(t)$ or the Holevo quantity $\chi(t)$, and test whether $\varepsilon_{\min}(t) \geq D(C_{\rm acc}(t))$ holds. Maintaining low distortion should require increasing reset or compression heat.

Protocol C: Analogue-horizon mode registration

Rationale. Analogue horizon systems (BEC, optical, acoustic, circuit) can realize tunable causal bottlenecks without requiring astrophysical black holes [14].

Observable. Define

$$C_{\rm reg}(g) = \begin{array}{l}\text{number of recoverable mode}\\ \text{distinctions at control parameter } g\end{array}, \quad (54)$$

and the reconstructability decay rate

$$\kappa_{\rm reg}(g) = -\frac{d}{dt}\log \mathcal{R}_g(t), \quad (55)$$

where $\mathcal{R}_g(t)$ is the recoverability of initial mode labels or boundary-mode information. Measure the steady-state entropy production $\dot{\Sigma}_{\rm ss}(g)$ near the horizon-like transition.

Expected signature. Near an effective horizon transition or capacity bottleneck, $dC_{\rm reg}/dg$ or $d\dot{\Sigma}_{\rm ss}/dg$ should show a kink or anomaly. This protocol is exploratory; the strongest near-term DT-specific physical signature remains the topological two-kink proposed for FDS-P3 [5].

CONSEQUENCES

No infinite local observer

A finite observer cannot be replaced by an ideal observer with infinite memory and infinite causal access without changing the physical problem. Such an idealization may be mathematically useful, but it erases the central constraints that make observation physical. In FDS, observer idealization is allowed only as a limiting model whose domain of validity must be stated.

Holography as boundary accounting

From the FDS perspective, holographic bounds can be interpreted as boundary accounting laws for accessible distinguishability. Interior processes may be numerous, but finite observers access and stabilize information through boundaries. When gravity becomes relevant, the boundary itself carries the ultimate information ceiling. The area law is therefore the natural limiting form of finite distinguishability accounting.

Coarse-graining is forced, not optional

When $R^{(\tau)}_{\min}(\varepsilon; \Psi)$ exceeds $C_{\rm acc}$, coarse-graining is not a methodological choice. It is forced by physical capacity.

Different observers may choose different coarse-grainings because they have different budgets, tasks, horizons, and error tolerances.

Externalization is bounded

Maps, instruments, laboratories, computer memory, archives, and environmental records can expand an observer's effective budget. But they do not remove the budget. They create a larger boundary whose records require stability, access, error correction, and thermodynamic maintenance. In gravitational regimes, even the enlarged boundary remains subject to area ceilings.

Relation to black-hole complementarity

FDS-T1 gives a neutral way to discuss why different observers may assign different accessible descriptions near horizons. The exterior observer's accessible distinction budget is tied to the horizon boundary, while an infalling observer's local patch differs. This paper does not solve the black-hole information problem. It supplies a finite-observer vocabulary for describing the difference between global unitary preservation, boundary encoding, and finite observer recoverability.

Bridge failure contract

The results of this paper are conditional on the validity domain of the physical bridge assumptions used: Bekenstein-type entropy bounds, holographic or covariant entropy ceilings, finite causal access, and Landauer-style update accounting. If future quantum-gravity physics discovers a regime in which Bekenstein or holographic bounds fail—for example through presently unknown topological sectors, non-area-like information storage, or a fundamentally different accounting of accessible physical information—then the corresponding FDS-T1 bridge claims must be revised or withdrawn within that regime. Such a failure would not by itself falsify the algebraic FDS core, which only asserts finite projection, boundary-relative capacity, update constraints, and deficit-induced exits at the formal level. It would falsify the specific physical embedding of those formal objects into the currently accepted gravitational information bounds. The layered failure contract is summarized in Table III.

FALSIFICATION CONDITIONS AND TESTS

Bridge-level falsification

The FDS-T1 bridge would be undermined by any of the following:

1. a finite-energy, finite-radius physical system reliably encoding more information than the Bekenstein bound permits under conditions where the bound is supposed to apply;
2. a horizon-bounded system with entropy exceeding the holographic ceiling under an appropriate covariant formulation;
3. a finite observer capable of registering, preserving, updating, and operationally using unbounded distinctions with bounded energy, bounded memory, bounded causal access, and bounded error correction;
4. a physically implemented logically irreversible erasure process violating the generalized Landauer lower bound under its stated thermodynamic conditions.

Failure of one bridge does not automatically destroy the FDS formal core. It localizes the failure to the physical interpretation or to the external physical theorem being used.

Operational tests

Direct Planck-scale tests of holographic bounds are not currently available. But FDS-T1 motivates lower-energy proxy tests.

Protocol 1 (Finite-budget observation test)**.** *To test a finite distinguishability budget in a controlled system:*

1. *specify the observer or apparatus boundary B;*
2. *estimate memory capacity C_{mem}, boundary capacity C_{bd}, channel capacity C_{chan}, and effective externalized capacity $C_{\mathrm{ext}}^{\mathrm{eff}}$;*
3. *estimate the update-throughput bound $I_{\mathrm{update}}^{\mathrm{max}}(t, t+\tau)$ from available free energy or allowable dissipation;*
4. *define a task family Ψ and distortion tolerance ε;*
5. *estimate $R_{\mathrm{min}}^{(\tau)}(\varepsilon; \Psi)$ using compression curves, rate-distortion lower bounds, Fisher information, or task-specific coding lower bounds;*
6. *test whether performance degrades, coarse-grains or compresses, externalizes, prunes, or fails as R_{min} crosses C_{acc}.*

Protocol 2 (Analogue-horizon budget test). *In an analogue horizon system, compare the number of recoverable, operationally distinguishable modes across a horizon-like boundary to the effective boundary area, noise temperature, and channel-access constraints. FDS-T1 predicts that recoverable distinction capacity should be boundary-limited and should degrade as access across the effective horizon becomes thermodynamically or causally unavailable.*

Expected signatures

FDS-T1 predicts the following qualitative signatures:

1. **Boundary-limited recoverability:** recoverable distinctions scale with access boundary capacity rather than naive interior volume when the boundary is the bottleneck.

2. **Budget crossing:** as task distinction demand crosses accessible capacity, systems should show sharp increases in compression, pruning, error, latency, or externalization.

3. **Observer-dependent coarse-graining:** different observers with different budgets should stabilize different partitions of the same underlying physical process.

4. **Thermodynamic update cost:** high-rate irreversible updating should show increased dissipation consistent with Landauer-style accounting when its assumptions hold.

LIMITATIONS

First, this paper does not derive the Bekenstein or holographic bounds from the distinction primitive alone. It imports them as physical bridge constraints.

Second, the exact coefficient 1/4 in $S = k_{\mathrm{B}} A/(4\ell_{\mathrm{P}}^2)$ is not derived. The paper uses the established Bekenstein–Hawking formula as the strongest known limiting case.

Third, finite distinguishability is not the same as finite Hilbert-space dimension. A system may have a very large or infinite mathematical state description while a finite observer has limited operational access.

Fourth, the accessible budget formula (25) is conservative and schematic. Real systems may require detailed modeling of noise, error correction, coupling, clocking, decoherence, control cost, and measurement back-action.

Fifth, not every boundary is a horizon. The paper distinguishes general distinguishability horizons from event horizons, apparent horizons, causal diamonds, and engineered analogue horizons.

Sixth, the framework is compatible with unitary microscopic dynamics. It concerns accessible records and finite observation, not necessarily fundamental non-unitarity.

CONCLUSION

Finite observers have finite distinguishability budgets. Established entropy bounds constrain not only storage but operationally accessible distinctions. When task-relevant demand exceeds accessible capacity, finite systems must coarse-grain, compress, externalize, prune, relax, or fail. The key empirical signature is budget crossing: as demand crosses accessible capacity, error, latency, compression, externalization, pruning, and heat should show measurable transitions.

This paper does not derive quantum gravity, general relativity, the Standard Model, or the 1/4 coefficient in the Bekenstein-Hawking entropy formula. Its contribution is narrower: to make finite distinguishability a precise, auditable, and testable bridge concept.

Future work includes bounded-memory reversible computation experiments, two-kink signature tests, finite-observer entropy bounds, time-energy-distinction speed limits, and externalization cost accounting.

ACKNOWLEDGMENTS

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APPENDIX A: Notation Summary

APPENDIX B: Claim Status and Failure Conditions

The status of each central claim and what would constitute falsification is given in Table III.

APPENDIX C: Minimal Model Parameters

The minimal computable model described above uses the following parameters:

- Fixed memory capacity C_M;
- Channel capacity $C_Y(t)$;
- Causal-boundary capacity $C_H(t) = C_0\,\eta(t)$ with sigmoid transition $\eta(t) = [1 + \exp((t - t_h)/\sigma)]^{-1}$;
- Thermodynamic budget parameters: free-energy supply rate $\dot{F}_{\text{avail}}$, temperature T;

TABLE II. FDS-T1 notation summary.

Symbol	**Meaning**
$\mathcal{O}$	finite observer or distinction-register
$\mathcal{D}$	observer-accessible region or causal patch
$\partial\mathcal{D}$	operational, causal, or horizon boundary of the patch
$\pi_{\mathcal{O}}$	finite distinction projection implemented by $\mathcal{O}$
$N_{\mathcal{O}}$	number of operationally distinguishable alternatives
$C_{\mathcal{O}}$	bit capacity $\log_2 N_{\mathcal{O}}$
C_{mem}	internal memory / record capacity
C_{bd}	accessible boundary information capacity
C_{chan}	channel-limited capacity
C_{stock}	stock (storage/state) capacity, minimum of the above
$C_{\mathrm{ext}}^{\mathrm{eff}}$	effective externalized capacity after indexing and verification costs
$I_{\mathrm{update}}^{\mathrm{max}}$	maximum irreversible update throughput (update budget ceiling)
C_{acc}	effective accessible capacity, $\min\{C_{\mathrm{stock}}, I_{\mathrm{update}}^{\mathrm{max}}\}$
$R_{\mathrm{min}}^{(\tau)}(\varepsilon;\Psi)$	minimal task-relevant rate-distortion demand
Δ_{FDS}	boundary-relative capacity deficit
$\chi(t)$	instantaneous crossing parameter
E	energy of bounded region
R	radius or size scale of bounded region
A	area of relevant boundary or horizon
ℓ_{P}	Planck length $\sqrt{\hbar G/c^3}$
S_{acc}	observer-relative accessible entropy
$\dot{Q}_{\mathrm{maint}}$	maintenance heat dissipation
$\dot{Q}_{\mathrm{phys}}$	non-informational physical dissipation

- Source model: Gaussian with variance σ_Z^2 under MSE distortion, or binary with probability p under Hamming distortion;
- Distortion tolerance ε;
- Update window τ.

All numerical values used in the demonstration are fixed in the reproducible code release. The model is deterministic and requires no random seeds.

Claim	**Status**	**Dependency**	**Failure condition**
Finite observer budget	Formal/physical bridge	finite record carrier; finite operational error tolerance	finite physical observer with unbounded reliable distinctions under bounded resources
Bekenstein distinguishability ceiling	Physical bridge	Bekenstein bound	bounded system exceeding Bekenstein info limit under applicable assumptions
Holographic distinguishability ceiling	Physical bridge	holographic/covariant entropy bound	horizon-bounded region exceeding appropriate area entropy ceiling
Boundary-relative capacity deficit	Formal conditional	rate-distortion demand; accessible capacity estimate	task succeeds at full fidelity despite demand exceeding all accessible capacity
Budget-exit classification	Conditional theorem	positive deficit; bounded resources	no coarse-graining, no externalization, no pruning, no task relaxation, no failure under persistent deficit
Observer-relative accessible entropy	Interpretive bridge	finite projection; physical record access	entropy assignment requiring no finite record carrier, boundary, or coarse-graining

TABLE III. Claim status and failure conditions for the FDS-T1 bridge.

APPENDIX D: Reproducibility Checklist

1. Code availability: all simulation code is publicly released.
2. Deterministic execution: no random seeds required; configuration files fix all parameters.
3. Figure reproduction: running `src/generate_figures.py` reproduces all figures and data tables.
4. Parameter documentation: all model parameters are documented in the code configuration.
5. Platform independence: the code runs on standard Python environments with common scientific libraries.

APPENDIX E: Boundary of Applicability

The FDS-T1 bridge claims in this paper apply under the following conditions:

1. The observer is a finite physical record-bearing system with bounded memory, bounded energy, bounded channel access, and bounded update rate.
2. The system operates under known gravitational entropy bounds (Bekenstein, holographic) where applicable.
3. The Landauer bound for logically irreversible operations holds under the stated thermodynamic conditions.
4. Causal access is finite and bounded by the speed of light.

The claims do not apply to:

- Infinite or idealized observers with unbounded resources.
- Regimes where Bekenstein or holographic bounds are known to fail or are superseded.
- Systems operating outside the validity domain of the Landauer principle for irreversible operations.

The layered failure contract is summarized below.

CODE AVAILABILITY

The simulation code used to generate Figs. 1–4 is available at https://github.com/yiningwu-research/Distinction-Theory/tree/main/models/fds_t1. All model parameters are fixed in the configuration; the code is deterministic and does not require random seeds. Running `src/generate_figures.py` reproduces all figures (PDF and PNG) and CSV data tables from the source code in a single pass.

TABLE IV. Layered failure contract for FDS-T1 bridge claims.

Layer	**Claim status**	**Failure consequence**
FDS-0 Core	Formal definitional system	Falsified only by internal incoherence or contradictory definitions
T1 Physical Bridge	Conditional embedding of FDS objects into Bekenstein, holographic, and Landauer bounds	Fails if imported constraints are violated or superseded in applicable regimes
Holographic embedding	Horizon-bounded accessible distinction scales with boundary area	Revised if future quantum gravity gives non-area information accounting
Maintenance bound	Irreversible surplus distinctions carry Landauer update cost	Revised if Landauer accounting fails under stated conditions

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