Capacity Deficit and Entropy Production in Active Finite Systems: A Generalized Dissipation Ledger for Boundary Maintenance

Yining Wu
Independent Researcher
yining.wu@alumni.upenn.edu

FDS-P5 develops a physical bridge between task-relative capacity deficit and entropy-production pressure in active finite distinction systems. The FDS formal core defines active finite systems as boundary-maintaining systems with finite representational capacity, finite update windows, and finite resource budgets. A capacity deficit $\Delta_\epsilon(\tau) > 0$ is not heat, entropy production, or thermodynamic irreversibility by itself. It is an information-theoretic shortfall between task-relevant rate-distortion demand and effective capacity. The central claim of this paper is narrower: when a physically realized active finite system continues boundary maintenance under sustained positive deficit, the required correction, refresh, repair, synchronization, externalization, pruning, or recovery must be accounted for somewhere in a coupled entropy or resource ledger. This revised version adds an explicit audit convention to avoid double counting, uses reservoir-indexed entropy-production notation, separates physical recovery entropy from functional loss, ties the correction information flow to rate-distortion deficit, defines positive-density deficit windows, and states a deficit-maintenance-dissipation impossibility triangle. Deterministic simulations illustrate deficit crossing, unique-channel ledger decomposition, pruning dissipation return on investment, coupled-system externalization, invariant compression, maintenance-failure regimes, and entropy-production hysteresis. The paper does not claim that every distinction dissipates heat, that capacity deficit is thermodynamic entropy, that all entropy production is Landauer erasure, or that the Second Law is newly derived. Its contribution is a finite-system dissipation ledger for boundary maintenance under positive capacity deficit.

Scope and Boundary of the Theory. This paper is a physical-bridge and normal-form paper. It does not claim that capacity deficit itself dissipates heat. It does not claim that every distinction, observation, or uncertainty carries a Landauer cost. It does not rederive the Second Law, and it does not assert a universal equality between information shortfall and thermodynamic entropy production. It claims only that under physical bridge assumptions, active finite systems that continue boundary maintenance under positive capacity deficit must pay through one or more accounting channels: irreversible update, refresh, repair, sensing, synchronization, externalization, pruning, task relaxation, or recovery after damage. Positive deficit by itself does not force dissipation if the system simply abandons the task, passively degrades, or accepts boundary loss.

Claim-status summary

Table I summarizes the central FDS-P5 claims, their epistemic status, and the conditions under which they should be weakened or rejected.

Keywords: capacity deficit; entropy production; information thermodynamics; boundary maintenance; active finite distinction systems; Landauer principle; housekeeping dissipation; pruning; externalization; invariant compression; finite capacity; stochastic thermodynamics; thermodynamic uncertainty relations.

INTRODUCTION

From capacity deficit to physical cost

The FDS formal core defines active finite distinction systems as finite-capacity systems that maintain boundaries through state-dependent updates under resource constraints [1]. Its central chain is

$$\text{finite capacity} \rightarrow \text{capacity deficit} \rightarrow \text{approximation} \rightarrow \text{irreversib} \tag{1}$$

FDS-P1 defined the physical accounting boundary between formal distinctions and physical records [2]. FDS-P2 studied bounded-memory reversible computation and the housekeeping burden of retained inverse information [3]. FDS-N1 translated active boundary maintenance into a complex-systems normal form with maintenance load, pruning, externalization, and Phase-C collapse [4]. FDS-P5 asks a more specific physical question: when capacity deficit persists and the system continues to maintain its boundary, where does the cost appear in the coupled entropy ledger?

The answer is not that a deficit is heat. A deficit is an information-theoretic mismatch. Heat, entropy production, or exported disorder appear only when a physically realized system senses, stores, corrects, erases, refreshes, repairs, synchronizes, externalizes, prunes, or recovers from damage in response to that mismatch.

TABLE I. Central FDS-P5 claims, epistemic status, and failure or demotion conditions.

Claim	Status	What would weaken or falsify it
Capacity deficit is a task-relative information shortfall, not thermodynamic entropy by itself.	Boundary statement	Not an empirical prediction; it separates formal rate-distortion demand from physical entropy production.
Sustained positive deficit plus continued boundary maintenance requires correction, externalization, task relaxation, or failure.	FDS-core derived	A finite active system maintains all pre-registered task distinctions at fixed tolerance despite $R^{(\tau)}_{\min}(\epsilon) > C_{\text{eff}}$ and no correction, loss, task relaxation, or external support.
Physical correction cycles induce audit channels through update, refresh, repair, synchronization, externalization, and transport.	Physical bridge	Controlled physical systems perform sustained correction, refresh, repair, sensing, and synchronization at zero coupled-system entropy production or resource cost.
Logical erasure contributes a Landauer-style entropy-production floor under bridge assumptions.	Physical bridge	Reliable logically irreversible physical erasure under stated thermodynamic assumptions violates the Landauer lower bound.
Housekeeping entropy production can persist even when logical erasure is zero.	Accounting claim	Boundary maintenance, refresh, clocking, sensing, and repair are cost-free whenever preimage erasure is zero.
Externalization shifts the entropy ledger across accounting boundaries.	Accounting bridge	External records impose no write, verification, retrieval, latency, synchronization, maintenance, or environmental cost.
Pruning and invariant compression can reduce future entropy-production pressure when task function is preserved.	Conditional bridge	No task-preserving simplification ever reduces refresh, repair, verification, synchronization, or maintenance cost.
Deficit crossing predicts measurable signatures in heat, resource use, latency, resets, error floor, externalization, or failure risk.	Testable normal-form	Positive deficit can be sustained with no measurable change in any physical, computational, or task-performance channel.

Why this is not simply Landauer

The Landauer term covers logically irreversible preimage loss. In the FDS core, logical erasure per update is

$$b_t = H(M_t \mid M_{t+1}, Y_t), \tag{2}$$

and the physical bridge gives an informational heat-rate floor

$$\dot{Q}_{\text{info}} \geq \frac{k_{\text{B}} T \ln 2}{\tau} H(M_t \mid M_{t+1}, Y_t), \tag{3}$$

or equivalently an entropy-production rate floor

$$\dot{\Sigma}_{\text{info}} \geq \frac{k_{\text{B}} \ln 2}{\tau} H(M_t \mid M_{t+1}, Y_t), \tag{4}$$

when entropy is measured in physical units. Equation (4) is essential but incomplete. An active boundary system can have low or zero logical erasure while still paying for refresh, clocking, sensing, isolation, synchronization, control, repair, transport, external verification, and maintenance of physical carriers. P5 therefore treats Landauer erasure as one ledger term inside a broader active-system entropy account.

Contribution

This paper contributes: (i) a generalized entropy-production audit ledger for active finite systems; (ii) a rate-distortion grounded correction-flow lower bound; (iii) a positive-density version of the deficit-driven entropy-pressure theorem; (iv) an anti-double-counting convention for ledger terms; (v) a multi-reservoir entropy-production notation; (vi) a separation between physical recovery entropy and functional boundary loss; (vii) pruning and invariant compression criteria for future entropy relief; (viii) engineering protocols for testing deficit crossings; and (ix) a handoff to later precision-dissipation and finite-memory Second-Law treatments.

CORE FDS INGREDIENTS

Active finite distinction systems

The FDS core object is

$$S = (X, E, B, M, Y, A, U, \pi, \ell, \Phi, \mathcal{P}, \tau), \tag{5}$$

where X is internal state, E environment, B boundary, M memory or model, Y observation channel, A action space, U update rule, π finite projection, ℓ boundary-maintenance loss, Φ resource budget, $\mathcal{P}$ perturbation or pruning family, and τ update timescale [1]. The timescale matters because information quantities become rates only when divided by an update window.

An active boundary requires nontrivial update and relevance to future boundary loss:

$$\mathbb{P}\{U(M_t, Y_t) \neq M_t\} > 0, \qquad \mathrm{I}(M_{t+1}; \ell_{t+k}) > 0. \quad (6)$$

For empirical systems, the stronger ablation test compares $do(U)$ with a frozen, randomized, or identity update channel.

Capacity deficit and effective capacity

Let $Z_t = \psi(E_t, B_t)$ be a pre-registered task-relevant statistic needed to maintain the boundary to tolerance ε over update window τ. Let

$$R_{\min}^{(\tau)}(\varepsilon; \Psi_t) = \inf_{\psi \in \Psi} R_{\psi(E,B)}^{(\tau)}(\varepsilon) \quad (7)$$

be the minimal task demand over an admissible family Ψ. For effective capacity $C_{\text{eff}}(t)$, define

$$\Delta_\epsilon(t) = R_{\min}^{(\tau)}(\varepsilon; \Psi_t) - C_{\text{eff}}(t). \quad (8)$$

When $\Delta_\epsilon > 0$, a finite internal model cannot maintain all task-relevant distinctions at the required fidelity without approximation, externalization, task relaxation, or failure.

In this paper, $\Delta_\epsilon(t)$ is measured in capacity units (bits or equivalent) per update window τ. Division by τ converts this window-level shortfall into a correction-flow rate in Eq. (17). If $R_{\min}$ is instead defined as a rate, the factor $1/\tau$ should be omitted.

Following N1, a useful operational decomposition is

$$C_{\text{eff}}^{P5} = C_{\text{int}}^{\text{eff}} + C_{\text{ext}}^{\text{eff}} - C_{\text{sync}} - C_{\text{verify}} - C_{\text{latency}} - C_{\text{resource}}, \quad (9)$$

with the internal bottleneck

$$C_{\text{int}}^{\text{eff}} = \min\{C_{\text{mem}}, C_{\text{chan}}, C_{\text{upd}}, C_{\text{res}}, C_{\text{timing}}, C_{\text{verify}}\}. \quad (10)$$

All penalty terms in Eq. (9) are expressed in capacity-equivalent units after conversion through the task-specific bottleneck. This prevents mixing raw bits, time, and energy without a conversion convention.

Resource budget and maintenance failure

A physically realized active finite system has a resource balance of the form

$$\Phi_{t+1} \leq \Phi_t + \int_t^{t+\tau} \dot{F}_{\text{in}}(s) ds - \int_t^{t+\tau} \dot{Q}_{\text{total}}(s) ds - D_\ell(\ell_t), \quad (11)$$

where D_ℓ is boundary-loss damage or functional repair burden. In the present paper, D_ℓ is tracked separately from physical entropy production unless a domain bridge specifies the dissipative recovery process that realizes it.

GENERALIZED ENTROPY-PRODUCTION LEDGER

Audit convention and no double counting

Definition 1 (Accounting-boundary entropy ledger). Let $\mathcal{A}$ be an accounting boundary and let $\mathcal{P}_\mathcal{A}$ be a pre-registered set of physical processes inside or coupled to that boundary. Define the audited physical entropy-production rate as

$$\dot{\Sigma}_{\text{phys}}^{\mathcal{A}} = \sum_{p \in \mathcal{P}_\mathcal{A}} \dot{\Sigma}(p), \quad (12)$$

where each physical process p is counted once. Named terms such as sensing, synchronization, externalization, and repair are audit classes, not automatically independent additive mechanisms. If a concrete implementation assigns the same physical process to more than one class, the ledger must either choose a unique channel assignment or include overlap-correction terms.

Equivalently, one may write a class-level decomposition

$$\dot{\Sigma}_{\text{phys}}^{\mathcal{A}} = \dot{\Sigma}_{\text{info}} + \dot{\Sigma}_{\text{hk}} + \dot{\Sigma}_{\text{meas}} + \dot{\Sigma}_{\text{ctrl}} + \dot{\Sigma}_{\text{sync}} + \dot{\Sigma}_{\text{ext}} + \dot{\Sigma}_{\text{rec}}, \quad (13)$$

with the audit convention that the right-hand side is a partition of process costs for the chosen model. The recovery term $\dot{\Sigma}_{\text{rec}}$ refers to physical recovery, repair after damage, dissipative reset, uncontrolled relaxation, or loss-induced resource expenditure. Failure itself, as a task-level event, is not automatically entropy production; pure functional loss is tracked by $D_\ell(\ell)$. Methods for inferring entropy production from observable transitions in partially accessible systems are directly relevant to implementing this ledger decomposition in practice [27, 29].

Multi-reservoir entropy-production notation

For a channel i coupled to reservoirs $r \in \mathcal{R}_i$, the stochastic-thermodynamic form is

$$\dot{\Sigma}_i = \dot{S}_i^{\text{sys}} + \sum_{r \in \mathcal{R}_i} \frac{\dot{Q}_{i,r}}{T_r} \geq 0, \quad (14)$$

with a dimensionless version obtained by dividing by k_B. The familiar single-temperature shorthand $T^{-1}\dot{Q}_i$ should be read as a special case or an illustrative proxy. P5 does not require a single universal bath temperature.

TABLE II. FDS-P5 entropy-production ledger. Terms are audit classes, not universal independent additive laws. Heat-rate expressions are single-reservoir shorthand for the multi-reservoir form in Eq. (14).

Term	Representative expression	Meaning
$\dot{\Sigma}_{\rm info}$	$\frac{k_{\rm B}\ln 2}{\tau}H(M_t \mid M_{t+1}, Y_t)$	Logical erasure, overwrite, irreversible compression
$\dot{\Sigma}_{\rm hk}$	$\sum_r \dot{Q}_{{\rm refresh},r}/T_r + \dot{Q}_{{\rm clock},r}/T_r$	Record maintenance and carrier upkeep
$\dot{\Sigma}_{\rm meas}$	$\sum_r(\dot{Q}_{{\rm sense},r} + \dot{Q}_{{\rm channel},r} + \dot{Q}_{{\rm record},r})/T_r$	Sensing, transmission, record formation
$\dot{\Sigma}_{\rm ctrl}$	$\sum_r(\dot{Q}_{{\rm act},r} + \dot{Q}_{{\rm repair},r})/T_r$	Action, repair, boundary control
$\dot{\Sigma}_{\rm sync}$	$\sum_r(\dot{Q}_{{\rm coord},r} + \dot{Q}_{{\rm verify},r} + \dot{Q}_{{\rm latency},r})/T_r$	Coordination, verification, timing overhead
$\dot{\Sigma}_{\rm ext}$	$\sum_r(\dot{Q}_{{\rm write},r} + \dot{Q}_{{\rm retrieve},r} + \dot{Q}^{\rm ext}_{{\rm maint},r})/T_r$	External records and scaffolds
$\dot{\Sigma}_{\rm rec}$	$\sum_r \dot{Q}_{{\rm recovery},r}/T_r$	Physical recovery, damage repair, dissipative reset, or uncontrolled relaxation after
$D_\ell(\ell)$	task-specific damage functional	Functional loss or resource damage, not entropy by itself

Erasure and housekeeping are distinct

A system may satisfy $H(M_t \mid M_{t+1}, Y_t) = 0$ for some update, yet still dissipate because it must keep records stable, clock reversible operations, synchronize subsystems, move carriers, correct errors, or hold a boundary far from equilibrium. Thus

$$\dot{\Sigma}_{\rm info} = 0 \not\Rightarrow \dot{\Sigma}^{\mathcal{A}}_{\rm phys} = 0. \tag{15}$$

This distinction continues the P1 boundary between logical projection and dissipative physical implementation [2], and the P2 distinction between reversible logging and finite-memory housekeeping [3]. The O1 observer-register formalism further shows that measurement incurs stabilization costs through dynamic bottlenecks [6], while O2 register-time analysis identifies synchronization and latency as irreducible overheads in finite-record systems [7]. Recent experimental and theoretical work on finite-time memory erasure further confirms that realistic correction costs deviate from universal linear scaling and require concrete physical models [28].

CAPACITY DEFICIT AS CORRECTION PRESSURE

Positive-density deficit windows

Definition 2 (Positive-density deficit). A process has sustained positive deficit over update windows when

$$\liminf_{n\to\infty} \frac{1}{n}\left|\{1 \le j \le n : \Delta_\epsilon(t_j) > 0\}\right| > 0. \tag{16}$$

In continuous time, replace the count by $T^{-1}\int_0^T \mathbf{1}_{\Delta_\epsilon(t)>0}dt$.

Correction information flow

Capacity deficit creates a pressure for correction, not a direct heat term. Since $\Delta_\epsilon(t)$ is a rate-distortion shortfall over an update window, the minimal capacity-equivalent correction flow needed to fill that shortfall satisfies

$$\dot{I}^{\rm min}_{\rm corr}(t) \ge \frac{[\Delta_\epsilon(t)]_+}{\tau}, \tag{17}$$

up to relief supplied by task relaxation, invariant compression, pruning, or externalization. More generally,

$$\dot{I}^{\rm min}_{\rm corr}(t) \ge \frac{[[\Delta_\epsilon(t)]_+ - \mathcal{E}_{\rm exit}(t)]_+}{\tau}, \tag{18}$$

where $\mathcal{E}_{\rm exit}(t)$ is the capacity-equivalent relief provided by admissible exit channels, all expressed in capacity-equivalent relief units per update window. A minimal decomposition is

$$\mathcal{E}_{\rm exit}(t) = \mathcal{E}_{\rm prune}(t) + \mathcal{E}_{\rm ext}(t) + \mathcal{E}_{\rm inv}(t) + \mathcal{E}_{\rm relax}(t) + \mathcal{E}_{\rm res}(t),$$

where $\mathcal{E}_{\rm prune}$ is pruning or compression that reduces effective task demand, $\mathcal{E}_{\rm ext}$ is externalization that supplies effective capacity, $\mathcal{E}_{\rm inv}$ is invariant compression that reduces maintenance load, $\mathcal{E}_{\rm relax}$ is task relaxation that lowers $R_{\rm min}$, and $\mathcal{E}_{\rm res}$ is resource expansion or channel upgrade that raises $C_{\rm eff}$.

Example 1 (Window-level deficit accounting)**.** *A finite sensing-and-control system has* $C_{\rm eff} = 40$ *bits per update window. A boundary-maintenance task initially requires* $R_{\rm min} = 30$ *bits per window and later rises to* $R_{\rm min} = 70$*. The capacity deficit is*

$$\Delta_\epsilon = 70 - 40 = 30$$

bits per window. If externalization supplies $\mathcal{E}_{\rm ext} = 20$ *bits per window and task-preserving pruning supplies* $\mathcal{E}_{\rm prune} = 5$*, the remaining correction burden is*

$$[\Delta_\epsilon - \mathcal{E}_{\rm ext} - \mathcal{E}_{\rm prune}]_+ = 5$$

bits per window. P5 does not say that these five bits are heat by themselves. It says that maintaining the boundary without relaxing the task requires this remaining burden to be paid through some physical correction, update, repair, synchronization, or coupled external ledger.

A physical correction cycle has entropy-production pressure

$$\dot{\Sigma}_{\text{corr}} \geq \alpha_{\text{upd}} \dot{I}_{\text{upd}} + \alpha_{\text{repair}} R_{\text{repair}} + \alpha_{\text{sync}} R_{\text{sync}}, \quad (19)$$

with implementation-dependent coefficients. Equation (19) is a normal-form inequality, not a fundamental constant law.

A stronger measurable form

In model classes where physical correction entropy is lower-bounded by a coefficient $\kappa > 0$ times correction flow, the integrated ledger obeys

$$\int_0^T \dot{\Sigma}^{\mathcal{A}}_{\text{phys}}(t) dt \geq \frac{\kappa}{\tau} \int_0^T \left[[\Delta_\epsilon(t)]_+ - \mathcal{E}_{\text{exit}}(t) \right]_+ dt. \quad (20)$$

This is not a universal thermodynamic identity. It is a measurable FDS-P5 bridge form: as deficit pressure rises, some combination of correction entropy, exit relief, task loss, or failure should become observable.

The coefficient κ has units of entropy per capacity unit. It is implementation-dependent and is not asserted to equal $k_B \ln 2$, except in erasure-limited correction channels dominated by logically irreversible reset or overwrite. In those cases one may write $\kappa = \eta_{\text{impl}} k_B \ln 2$, where η_{impl} is an implementation efficiency factor capturing carrier dissipation, refresh overhead, and channel losses. This parametrization is a toy model for erasure-dominated channels and should not be read as a universal law.

Deficit crossing signatures

When Δ_ϵ crosses zero and task tolerance is held fixed, P5 predicts at least one of the following signatures: increased heat or resource use, reset bursts, refresh activity, latency, synchronization overhead, error floor, pruning, externalization, task relaxation, or failure risk. If the system accepts loss or abandons the task, positive deficit need not produce correction entropy; it produces boundary failure or task relaxation instead.

DEFICIT-DRIVEN ENTROPY-PRODUCTION PRESSURE THEOREM

Theorem 1 (Deficit-driven entropy-production pressure)**.** *Consider a physically realized active-boundary FDS over task window τ. Suppose: (i) $\Delta_\epsilon(t) > 0$ persists over a positive-density set of update windows; (ii) boundary-maintenance loss must satisfy $\ell(t) \leq \ell_c$; (iii) the task is not relaxed; (iv) externalization, invariant compression, pruning relief, and resource expansion are bounded; and (v) physical correction channels have nonzero coupled-system entropy-production or resource cost. Then the system cannot maintain the same boundary task indefinitely at zero coupled-system entropy-production cost. It must enter at least one channel: increased physical entropy production, exported entropy production, pruning, invariant compression, task relaxation, resource expansion, passive degradation, or collapse.*

Proof. Positive capacity deficit means the current effective capacity is insufficient to encode the least demanding admissible boundary statistic to tolerance ϵ over the update window. If the system maintains $\ell \leq \ell_c$ without relaxing the task, it must supply additional effective distinctions through correction, approximation update, repair, externalization, pruning, compression, or resource expansion. By hypothesis, the available exit relief is bounded and each physically realized correction or relocation channel has nonzero coupled-system entropy-production or resource cost. If those costs remain inside the finite resource envelope, the system persists at increased or redistributed entropy production. If they exceed the envelope and no exit channel is used, Eq. (11) drives the system toward maintenance failure. Therefore zero-cost indefinite maintenance under sustained deficit is impossible under the stated assumptions. □

Corollary 1 (Deficit-maintenance-dissipation impossibility triangle)**.** *An active finite system cannot jointly satisfy*

$$\Delta_\epsilon > 0 \textit{ sustained}, \qquad \ell \leq \ell_c \textit{ unchanged}, \qquad \dot{\Sigma}^{\mathcal{A}}_{\text{phys}} = 0 \quad (21)$$

under finite resources and nonzero physical correction costs. At least one term must change: the deficit is reduced, the task is relaxed, entropy is produced or exported, resources expand, passive degradation is accepted, or the boundary-maintenance regime fails.

Deficit–maintenance–dissipation impossibility triangle. Under the P5 physical bridge assumptions, sustained positive capacity deficit, fixed boundary-maintenance tolerance, and zero coupled-system entropy or resource cost cannot persist indefinitely:

$$\Delta_\epsilon > 0 \text{ sustained}, \qquad \ell \leq \ell_c \text{ unchanged}, \qquad \dot{\Sigma}^{\mathcal{A}}_{\text{phys}} = 0$$

are jointly impossible without pruning, compression, externalization, task relaxation, invariant stabilization, resource expansion, or collapse.

Remark 1. *The theorem does not assert that every deficit produces the same heat rate. It asserts that sustained active maintenance under positive deficit must appear somewhere in the coupled physical ledger or be replaced by an exit channel.*

PRUNING, COMPRESSION, AND EXTERNALIZATION

Pruning as dissipation return on investment

Pruning may incur a one-time cost $\Sigma_{\rm prune}$ while reducing future housekeeping and synchronization costs. Over horizon H, define a dissipation return on investment

$$\mathrm{ROI}_{\Sigma}^{\rm prune}(H) = \frac{\int_t^{t+H}\left(\dot{\Sigma}_{\rm tot}^{\rm no\ prune}(s) - \dot{\Sigma}_{\rm tot}^{\rm prune}(s)\right) ds - \Sigma_{\rm prune}}{\Sigma_{\rm prune} + \lambda D_{\rm task}}. \tag{22}$$

A task-preserving pruning operation is favorable when $\mathrm{ROI}_{\Sigma}^{\rm prune}(H) > 0$, equivalently

$$\int_t^{t+H} \Delta\dot{\Sigma}_{\rm future}(s) ds > \Sigma_{\rm prune} + \lambda D_{\rm task}. \tag{23}$$

This expresses active forgetting, model distillation, cleanup, or biological self-repair as an entropy-management policy, not merely a failure. If task-relevant distinctions are destroyed, $D_{\rm task}$ increases and pruning becomes over-pruning.

Invariant compression

Let $q : X \to Q$ be a quotient map and $R_A = \bar{R}_A \circ q$ an identity predicate preserved under admissible perturbations. If

$$q \circ P_i = q \quad \forall P_i \in \mathcal{P}, \tag{24}$$

then the identity carried by R_A may be maintained through the lower-dimensional quotient q rather than all microstate distinctions. Invariant compression lowers entropy-production pressure when

$$\dot{\Sigma}_{\rm hk}^{q} + \dot{\Sigma}_{\rm verify}^{q} + \lambda D_{\rm task}^{q} < \dot{\Sigma}_{\rm hk}^{X} + \dot{\Sigma}_{\rm verify}^{X}. \tag{25}$$

This links P5 to Phase-B invariant selection in FDS-T3 [5] and invariant-supported persistence in the FDS core [1].

Externalization is entropy relocation

Externalization changes the accounting boundary:

$$\mathcal{A}_{\rm local} \to \mathcal{A}_{\rm coupled} = \mathcal{A}_{\rm local} \cup \mathcal{A}_{\rm ext}. \tag{26}$$

Local entropy production may fall, but the coupled ledger includes write, verification, retrieval, latency, synchronization, carrier maintenance, and environmental noise:

$$\dot{\Sigma}_{\rm phys}^{\rm coupled} = \dot{\Sigma}_{\rm local} + \dot{\Sigma}_{\rm write} + \dot{\Sigma}_{\rm verify} + \dot{\Sigma}_{\rm retrieve} + \dot{\Sigma}_{\rm sync} + \dot{\Sigma}_{\rm env}. \tag{27}$$

Externalization therefore exports or restructures entropy production; it does not abolish the ledger. If the environment is itself a finite record-bearing system, excessive externalization can create verification, indexing, latency, or clogging costs, as modeled in N1 [4].

PRECISION-DISSIPATION AND SECOND-LAW HANDOFFS

P5 defines the entropy ledger under capacity deficit. It does not yet prove a thermodynamic uncertainty relation for boundary maintenance. However, the natural next step is a precision-dissipation bound linking fluctuations of boundary loss, repair currents, or record-update currents to entropy production. Standard thermodynamic uncertainty relations show that precision of currents in nonequilibrium systems is constrained by dissipation [24, 25], and recent entropic uncertainty relations connect observable Shannon entropy to entropy production [30]. In the FDS sequence, such precision-cost questions belong to P6.

P5 also hands off to O3. P1 defines physical carriers and erasure maps; P2 defines finite-memory reversible reuse; N1 defines maintenance load and collapse regimes; P5 defines the deficit-driven entropy ledger. O3 can then study how finite memory, coarse-grained irreversible update, record reuse, and boundary-maintenance cost form an operational Second-Law channel for active finite systems.

NUMERICAL MODELS AND SIMULATIONS

The simulations are deterministic synthetic demonstrations. They are not fits to physical memory devices, biological organisms, computers, or human-subject data. They provide reproducible diagrams for the P5 normal form. All figures and CSV outputs are generated by `code/generate_results.py`.

EXPERIMENTAL AND ENGINEERING PROTOCOLS

Protocol 1 (Deficit-crossing protocol)**.** *Gradually increase task demand or reduce effective capacity so that* $R_{\rm min}^{(\tau)}(\epsilon;\Psi)$ *crosses* $C_{\rm eff}$*. Measure heat or power, refresh activity, reset events, latency, error floor, pruning, externalization, task relaxation, and boundary loss. Whenever possible,* $R_{\rm min}^{(\tau)}(\epsilon;\Psi)$ *and* $C_{\rm eff}$ *should be estimated or preregistered independently before observing heat, power, latency, reset, or failure signatures. This prevents post-hoc definition of deficit from the cost signal itself.*

Protocol 2 (Ledger decomposition protocol)**.** *For a fixed task, assign physical processes to unique audit chan-*

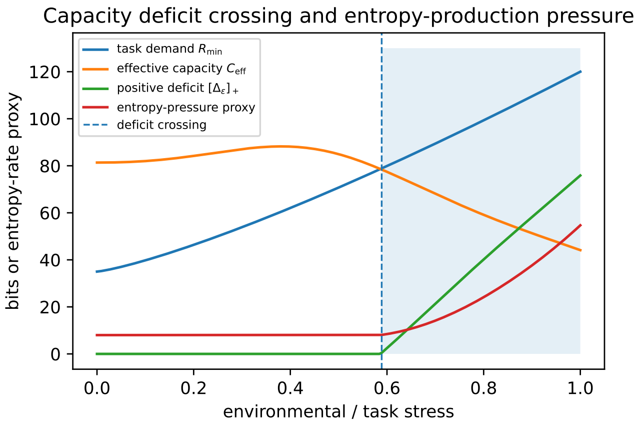

FIG. 1. Capacity deficit crossing and entropy-production pressure. Task demand rises above effective capacity at the dashed line; the shaded region marks $\Delta_\epsilon > 0$. Positive deficit increases the correction-flow and entropy-pressure proxies.

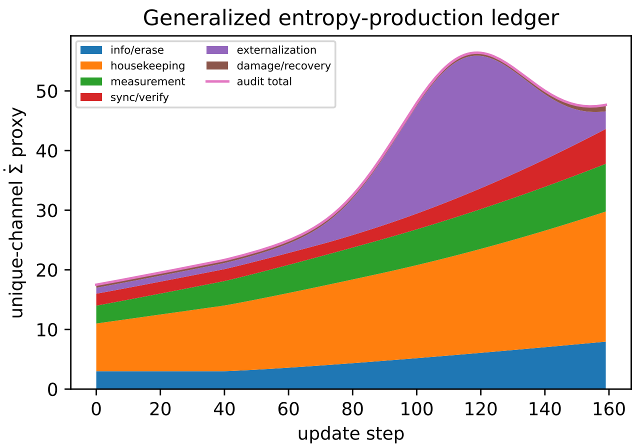

FIG. 2. Generalized entropy-production ledger. Stacked areas are illustrative unique-channel assignments under an audit convention. They should not be read as independent universal additive laws without a concrete process partition. The audit-total curve is a boundary-audited sum under the non-overlap convention, not an additional physical channel.

nels: information erasure, housekeeping, sensing, control, synchronization, externalization, and recovery. Verify that the same physical process is not double-counted across channels.

Protocol 3 (Pruning ROI protocol)**.** *Compare no pruning, task-preserving pruning, and over-pruning. Measure one-time pruning cost, future housekeeping reduction, task loss, and the finite-horizon quantity in Eq.* (22).

Protocol 4 (Externalization full-boundary audit)**.** *Move records or functions outside the local boundary and measure local savings, write cost, verification cost, retrieval cost, synchronization, latency, and environmental noise. Local savings without coupled-system savings count as relocation, not elimination.*

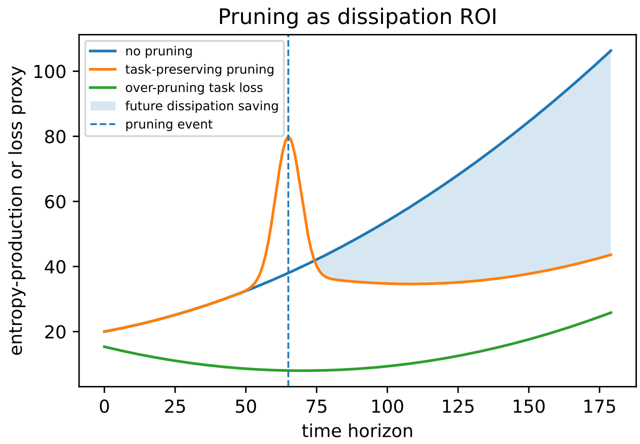

FIG. 3. Pruning as dissipation ROI. A one-time pruning event can reduce future entropy production; the shaded area illustrates future dissipation saving. Over-pruning is represented separately as task-loss penalty.

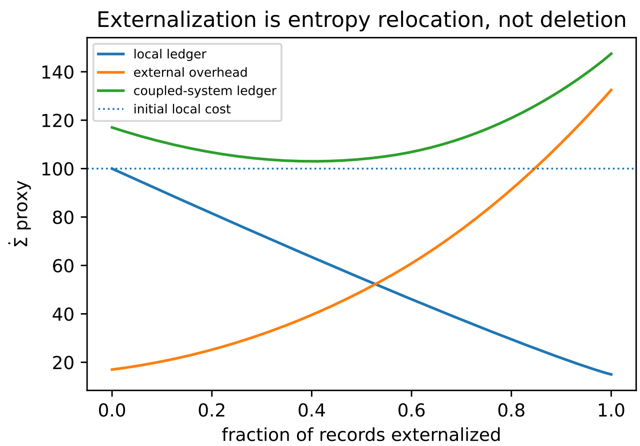

FIG. 4. Externalization is entropy relocation, not deletion. Local cost can fall while external overhead and coupled-system entropy production rise because write, verification, retrieval, synchronization, and environmental costs become active.

Protocol 5 (Hysteresis protocol)**.** *Increase deficit above the crossing threshold, then lower it again. Test whether entropy-production, resource use, or error curves return to the original path or retain a gap from erased records, damage, or lost invariant support.*

RELATION TO EXISTING WORK

Landauer's principle links logical irreversibility and heat generation under physical assumptions [11, 12, 14]. Modern information thermodynamics extends this connection to measurement, feedback, stochastic trajecto-

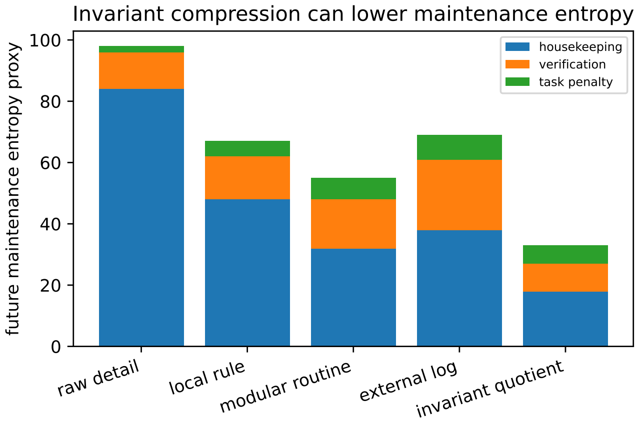

FIG. 5. Invariant compression can reduce maintenance entropy when the quotient retains task function with lower housekeeping and verification burden.

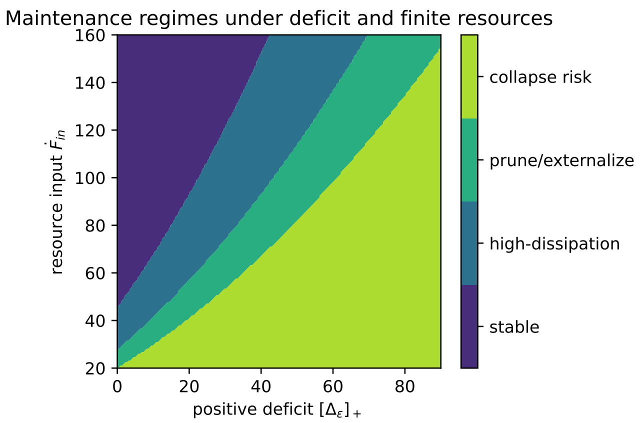

FIG. 6. Maintenance regimes under capacity deficit and finite resource input. Stable regimes satisfy the ledger within the resource envelope; high-dissipation regimes maintain the task by paying more entropy production; exit regimes require pruning or externalization; collapse risk appears when the envelope is exceeded.

ries, and nonequilibrium processes [15–18]. Stochastic thermodynamics of computation emphasizes finite-time, finite-bath, correlated, non-Markovian, and architectural constraints [20–23]. Thermodynamic uncertainty relations link dissipated entropy to precision constraints in nonequilibrium currents [24, 25].

P5 is not a replacement for those theories. It is an FDS bridge: it organizes the entropy-producing channels that appear when a finite active system maintains a boundary under capacity deficit. Compared with the free-energy principle and active inference [26, 31], P5 focuses less on variational objective functions and more on explicit task-relative capacity deficit, accounting boundaries, and entropy-production channels. Compared with dissipative-structure and self-organization theories, P5

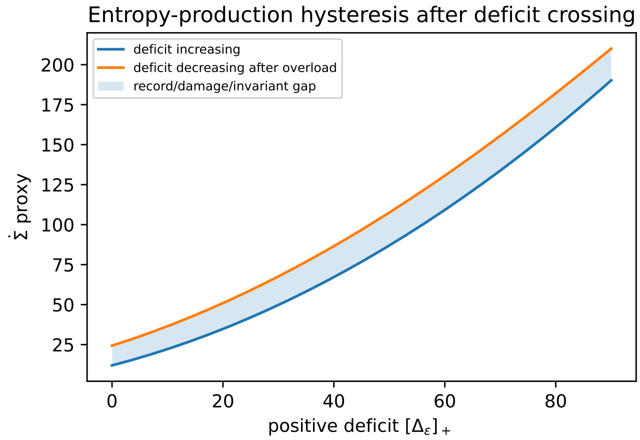

FIG. 7. Entropy-production hysteresis after deficit crossing. The gap between increasing and decreasing curves represents path-dependent record loss, accumulated damage, or destroyed invariant support; it is not asserted as a universal hysteresis law.

focuses on maintained distinctions and finite update ledgers rather than order formation alone.

LIMITATIONS AND FALSIFICATION

P5 does not claim that capacity deficit itself is heat, that every bit of uncertainty costs $k_B T \ln 2$, that all entropy production is Landauer erasure, or that all active systems share the same cost function. It also does not claim that pruning always helps, externalization always helps, or invariant compression is always available.

The strong version of P5 would be weakened or rejected by any of the following:

1. physically implemented logically irreversible erasure reliably violates the Landauer bridge under its stated assumptions;
2. active finite systems maintain sustained positive deficit and fixed task tolerance with no correction, externalization, pruning, task relaxation, loss increase, or physical cost;
3. correction, refresh, repair, sensing, synchronization, or externalization channels show no entropy-production or resource burden in controlled physical implementations;
4. externalization reduces local cost while imposing no coupled-system write, verification, retrieval, maintenance, latency, or environmental cost;
5. task-preserving pruning or invariant compression never reduces future maintenance load in any controlled system;

6. deficit crossing produces no measurable heat, energy, latency, reset, error, externalization, pruning, or failure signature.

CONCLUSION

FDS-P5 connects task-relative capacity deficit to entropy-production pressure in physically realized active finite systems. The compact statement is:

> Capacity deficit does not dissipate heat by itself. But when a physically realized active finite system tries to maintain a boundary under sustained positive deficit, it must pay somewhere in the coupled ledger through entropy-producing correction, housekeeping, synchronization, externalization, pruning, task relaxation, recovery, or failure.

This places P5 between the physical accounting papers P1/P2, the complex-systems bridge N1, and the later O3 treatment of boundary maintenance and the Second Law under finite memory. The central object is not abstract entropy. It is a finite system trying to keep a boundary usable while its task demands exceed its available distinctions.

Notation summary

Symbol	Meaning
B	maintained boundary or interface
M	memory, model, or organizational state
Y	observation channel
U	internal update rule
ℓ	boundary-maintenance loss
Φ	finite resource or free-energy budget
τ	update window or timescale
$R^{(\tau)}_{\min}(\epsilon;\Psi)$	task-relevant rate-distortion demand
$C_{\rm eff}$	effective task-available capacity
Δ_ϵ	capacity deficit $R^{(\tau)}_{\min} - C_{\rm eff}$
$\dot{I}_{\rm corr}$	correction information flow
$H(M_t\|M_{t+1}, Y_t)$	logical erasure per update
$\dot{\Sigma}^{\mathcal{A}}_{\rm phys}$	audited physical entropy-production rate
$\dot{\Sigma}_{\rm info}$	informational erasure entropy-production rate
$\dot{\Sigma}_{\rm hk}$	housekeeping entropy-production rate
$\dot{\Sigma}_{\rm ext}$	externalization entropy-production rate
$\dot{\Sigma}_{\rm rec}$	recovery or repair entropy-production rate
$D_\ell(\ell)$	functional boundary-loss damage
q	invariant quotient or compression map

Simulation parameters

The simulations use deterministic arrays with fixed synthetic parameters in `code/generate_results.py`. Figure 1 varies task stress and effective capacity. Figure 2 decomposes the entropy ledger using unique audit channels. Figure 3 compares no pruning, task-preserving pruning, and over-pruning. Figure 4 varies externalization fraction and plots local versus coupled cost. Figure 5 compares representation strategies. Figure 6 classifies deficit-resource regimes. Figure 7 illustrates hysteresis after overload. No proprietary, biological, organizational, human-subject, or device data are used.

Reproducibility checklist

1. Code availability: all simulation code is included in the replication package.
2. Deterministic execution: the code uses fixed synthetic parameters and deterministic arrays.
3. Figure reproduction: run `python code/generate_results.py`; the script regenerates all figures and CSV outputs.
4. Data status: all numerical outputs are synthetic demonstrations generated from the stated model.
5. Platform independence: the code uses standard Python scientific libraries.

Code availability

The simulation code used to generate Figs. 1–7 is included in the accompanying replication package under `code/generate_results.py`. Running the script regenerates all figures (PDF and PNG) and CSV tables in a single pass.

AI assistance disclosure

AI-assisted tools were used for language polishing, structural feedback, LaTeX drafting support, and code-debugging assistance. The author reviewed and edited all content and remains responsible for all claims, references, simulations, and conclusions. No AI system is listed as an author.

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