

# Boundary-Access Holonomy and the Quantum Reversible Quotient A Finite-Distinction Framework for the Zero-Record-Holonomy Sector

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(Dated: June 2026)

Standard finite-dimensional quantum mechanics is usually formulated after its symmetric structure has already been assumed: states live in a complex projective Hilbert space, reversible dynamics is unitary, and distinguishability is symmetric inside the closed-system formalism. FDS-Q0 asks a prior question: why should physical distinguishability become symmetric at all? The answer proposed here is that standard quantum mechanics is the zero-holonomy reversible quotient of a more primitive asymmetric finite-boundary access dynamics. A physical distinction is first directed, boundary-relative, and capacity-limited: a system may export, preserve, erase, or recover a distinction differently across different boundaries. The central object is the boundary-access holonomy  $\mathcal{H}_\partial$ , an operational obstruction to closing a directed access loop through system, apparatus, environment, record, and recovery channels. This version strengthens the Q0 proposal in three ways. First, it defines a restricted access category with finite quantum systems, finite record registers, quantum instruments, record channels, and recovery maps, and proves a restricted zero-holonomy quotient-consistency theorem. Second, it separates coherent holonomy, record holonomy, and leakage holonomy, preventing geometric phases from being conflated with measurement records. Third, it supplies two minimal operational models: a Mach-Zehnder stable-record test in which recoverable visibility depends on stable accessible record holonomy after controlling ordinary overlap, and a QEC directed-leakage model in which logical failure depends on uncorrectable leakage information after controlling average physical error. The paper does not replace standard quantum mechanics, deny unitary evolution, prove objective collapse, derive quantum field theory, or complete the Born-rule proof. It supplies an upstream FDS architecture and a testable normal-form program:  $\mathcal{H}_\partial = 0$  gives the reversible Hilbert sector, while  $\mathcal{H}_\partial \neq 0$  gives record, decoherence, measurement, and maintenance sectors.

**Scope and Claim Status.** Q0 formulates reversible quantum mechanics as a quotient problem over finite boundary access. The central structural claim is that standard Hilbert-space quantum mechanics is the zero-holonomy reversible quotient of asymmetric finite boundary access. Q0 does not replace Q1 or Q2. Q1 studies when an interaction becomes an accessible record; Q2 studies how fragile logical quantum distinctions are maintained. Q0 supplies the upstream layer: why a symmetric quantum sector exists at all. Q0 is not a collapse theory, not a hidden-variable theory, not a denial of standard quantum predictions, and not a completed derivation of all quantum mechanics.

## Claim-status summary

**Keywords:** finite distinction systems; quantum foundations; boundary access; holonomy; measurement; decoherence; Wigner's friend; quantum error correction; Born rule; record capacity; reversible quotient; operational facts.

## INTRODUCTION

### The missing upstream question

Quantum mechanics is often introduced through a formal package: complex Hilbert spaces, density operators, observables, unitary evolution, tensor products, and the Born rule. This package works. Q0 does not dispute its empirical success. The question is upstream: why is the reversible quantum sector symmetric at all?

Inside standard finite-dimensional quantum mechanics, pure-state distinguishability may be written as

$$K_Q(\psi, \phi) = 1 - |\langle \psi | \phi \rangle|^2, \quad (1)$$

which is symmetric by construction. However, physical access to a distinction is rarely symmetric before the closed-system quotient is imposed. A system can imprint a which-path mark on an environment; the environment may stabilize the mark; an observer may read it; and reversal may require active erasure. The forward and reverse accessibility structures are not equivalent.

### Q0 thesis

Q0 proposes that standard quantum mechanics is not the primitive theory of all physical distinction. It is the

TABLE I. Central FDS-Q0 claims, status, and demotion conditions.

Claim	Status	Demotion or failure condition
Physical distinguishability is first directed boundary access	FDS primitive / definition	All operational record production, erasure, and recovery are symmetric without boundary cost or accessibility structure
Zero record holonomy makes the reversible quotient well-defined; in the restricted quantum-access category the induced kernel is symmetric	Restricted quotient-consistency theorem in Sec.	A closed, record-neutral, zero-holonomy sector fails to admit any well-defined symmetric quotient kernel
Hilbert QM is the finite-capacity zero-holonomy sector	Conditional theorem target	A non-Hilbert reciprocal finite-capacity quotient satisfies reversible dynamics, composition stability, and no hidden record asymmetry
Born weights are the no-record-capacity-arbitrage rule	Open theorem target	A non-Born rule satisfies basis neutrality, coarse-graining, composition, and finite record-capacity no-arbitrage
Decoherence irreversibility tracks stable records, not mere entanglement alone	Operational prediction	Stable record accessibility has no independent predictive role for recovery once ordinary overlap and noise metrics are controlled
QEC overhead tracks directed leakage holonomy	Q0/Q2 bridge	Directed leakage has no independent predictive value for logical failure after matched-fidelity and matched-syndrome controls

zero-holonomy quotient of asymmetric finite boundary access:

$$\begin{aligned} \mathcal{H}_\partial = 0 &\Rightarrow \text{reversible Hilbert sector,} \\ \mathcal{H}_\partial \neq 0 &\Rightarrow \text{record/decoherence/maintenance sectors.} \end{aligned} \quad (2)$$

The important point is not merely that measurements are irreversible. The important point is that reversible quantum mechanics itself is recovered only after quotienting away directed boundary access, stable records, uncorrectable leakage, and irreversible update.

### Relation to FDS Quantum Papers

Q0 is upstream of the FDS quantum papers. O1 identifies observation with finite record formation. O2 identifies operational time with irreversible finite distinction update. Q1 studies Wigner-friend record promotion. Q2 studies protected logical quantum distinctions and QEC maintenance ledgers. Q0 asks the prior question: why does a symmetric reversible quantum sector exist at all?

This relation is organizational, not a logical premise. The restricted theorem and operational models in Q0 depend only on finite boundary access, record holonomy, leakage holonomy, and the declared access category.

### FINITE BOUNDARY-ACCESS SYSTEMS

**Definition 1** (Finite boundary-access system). *A finite boundary-access system is a tuple*

$$\mathcal{Q} = (X, \partial X, C, \mathcal{I}, \mathcal{A}, \Pi, \mathcal{R}), \quad (3)$$

where  $X$  is a set of potential distinctions,  $\partial X$  is an access boundary,  $C$  is a finite capacity budget,  $\mathcal{I}$  is a class of admissible boundary interactions or instruments,  $\mathcal{A}$  is a directed access distinguishability functional,  $\Pi$  is a quotient/projection operation, and  $\mathcal{R}$  is a record-stabilization map.

**Definition 2** (Operational directed access). *For an instrument  $\mathcal{I}$ , define directed access by record distributions:*

$$\mathcal{A}_\partial^\mathcal{I}(x \rightarrow y) = D(P_\partial^\mathcal{I}(R|x), P_\partial^\mathcal{I}(R|y)), \quad (4)$$

where  $R$  is a boundary-accessible record and  $D$  is an admissible operational distinguishability functional, such as total variation distance, Chernoff distinguishability, trace-distance-induced distinguishability, or a task-specific discrimination distance.

**Remark 1** (Why no universal  $D$  is fixed here). *Q0 does not need a universal distance functional at the outset. Different experimental protocols operationalize distinguishability differently. The locked claim is not a special metric formula; it is that stable record structure and directed recovery obstruction can be operationally separated from mere closed-system overlap.*

**Definition 3** (Access asymmetry). *The directed access asymmetry is*

$$\Delta_\partial^\mathcal{I}(x, y) = \mathcal{A}_\partial^\mathcal{I}(x \rightarrow y) - \mathcal{A}_\partial^\mathcal{I}(y \rightarrow x). \quad (5)$$

*Reciprocal access for a pair means  $\Delta_\partial^\mathcal{I}(x, y) = 0$ .*

**Definition 4** (Finite maintained distinction). *A distinction is physical only if it is maintainable under finite capacity, finite error, finite time, finite memory, finite access, and finite update resources. This is stronger than a formal label difference.*

TABLE II. Three holonomy sectors.

Sector	Meaning	Example
Coherent	reversible phase transport, no record	Berry phase
Record	stable record export	which-path record
Leakage	logical distinction leaves correctable sector	QEC leakage

## BOUNDARY-ACCESS HOLONOMY

### Access loops

Consider an access loop

$$\gamma : S \rightarrow A \rightarrow E \rightarrow R \rightarrow S, \quad (6)$$

where  $S$  is a system,  $A$  an apparatus,  $E$  an environment,  $R$  a record layer, and the final arrow asks whether the original distinction can be recovered at the system boundary.

**Definition 5** (Boundary-access holonomy). *Boundary-access holonomy is the obstruction to recovering a distinction after a directed access loop. Schematically,*

$$\mathcal{H}_\partial(\gamma) = \mathcal{A}_{S \leftarrow R} \circ \mathcal{A}_{R \leftarrow E} \circ \mathcal{A}_{E \leftarrow A} \circ \mathcal{A}_{A \leftarrow S} - \text{id}. \quad (7)$$

Operationally, a holonomy norm may be represented by a record-recovery distance

$$\|\mathcal{H}_\partial(\gamma)\|_{\text{rec}} = D_{\text{rec}}(P_{\text{fwd}}(R), P_{\text{rec}}(R)), \quad (8)$$

by a nonzero erasure cost, by loss of interference visibility, by stable environment-accessible mutual information, or by uncorrectable logical leakage.

### Three holonomy sectors

The word ‘‘holonomy’’ is overloaded in physics. Q0 therefore separates three sectors.

**Definition 6** (Coherent holonomy). *Coherent holonomy is reversible transport holonomy, such as Berry phase or gauge phase, that changes phases but does not by itself stabilize a boundary record. It belongs to the reversible quantum sector unless coupled to record formation.*

**Definition 7** (Record holonomy). *Record holonomy is nonzero access holonomy in which a distinction is exported into a stable boundary-accessible record. This is the measurement/decoherence/fact-formation sector.*

**Definition 8** (Leakage holonomy). *Leakage holonomy is nonzero access holonomy in which a protected logical distinction leaks into an uncorrectable boundary or environment sector. This is the QEC maintenance-failure sector.*

## RESTRICTED ACCESS CATEGORY AND REVERSIBLE QUOTIENT

The general Q0 program is broad. To avoid leaving the central object schematic, this section gives a restricted theorem in a concrete access category.

**Definition 9** (Restricted access category  $\mathcal{C}_{\text{acc}}$ ). *Let  $\mathcal{C}_{\text{acc}}$  have objects  $(\mathcal{H}_S, \mathcal{M}_R)$ , where  $\mathcal{H}_S$  is a finite-dimensional quantum system and  $\mathcal{M}_R$  is a finite classical record register. Morphisms are generated by: (i) CPTP maps on finite systems, (ii) quantum instruments  $\{\mathcal{E}_r\}_r$ , (iii) classical stochastic record channels, and (iv) recovery maps  $\mathcal{V}$  from record/system outputs back to system states. The operational distinguishability is either trace distance for quantum outputs or total variation distance for classical records.*

**Definition 10** (Restricted zero record holonomy). *A pair  $x, y$  has  $\epsilon$ -zero record holonomy along an access loop  $\gamma$  if there exists an admissible recovery map  $\mathcal{V}_\gamma$  such that both the forward record distributions and the recovered state distinguishabilities close within  $\epsilon$ :*

$$D_{\text{TV}}(P_{\text{fwd}}(R|x), P_{\text{rec}}(R|x)) \leq \epsilon, \quad (9)$$

$$D_{\text{TV}}(P_{\text{fwd}}(R|y), P_{\text{rec}}(R|y)) \leq \epsilon, \quad (10)$$

$$|D_{\text{tr}}(\rho_x, \rho_y) - D_{\text{tr}}(\mathcal{V}_\gamma \rho'_x, \mathcal{V}_\gamma \rho'_y)| \leq \epsilon. \quad (11)$$

The exact-zero case is  $\epsilon = 0$ .

**Definition 11** (Reversible quotient relation). *Define  $x \sim_{\text{rev}} x'$  when every admissible record-neutral access test gives identical recovered distinguishability between  $x$  and any third distinction up to the declared tolerance. Let  $[x]$  be the equivalence class.*

**Theorem 1** (Restricted zero-holonomy quotient-consistency theorem). *In  $\mathcal{C}_{\text{acc}}$ , suppose a sector is exact-zero record-holonomy, record-neutral, and closed under admissible recovery maps. Define*

$$K_{\text{rev}}([x], [y]) = D_{\text{tr}}(\rho_x, \rho_y) \quad (12)$$

on recovered representatives. Then  $K_{\text{rev}}$  is well-defined on reversible quotient classes and symmetric:

$$K_{\text{rev}}([x], [y]) = K_{\text{rev}}([y], [x]). \quad (13)$$

For  $\epsilon$ -zero holonomy, symmetry and representative-independence hold up to  $O(\epsilon)$ .

*Proof.* Representative-independence follows from the definition of  $\sim_{\text{rev}}$ : if  $x \sim_{\text{rev}} x'$ , all admissible record-neutral recovered distinguishability tests agree on  $x$  and  $x'$ , hence  $D_{\text{tr}}(\rho_x, \rho_y) = D_{\text{tr}}(\rho_{x'}, \rho_y)$  for all recovered representatives in the exact case. Symmetry follows from symmetry of trace distance on the recovered quantum sector,

$$D_{\text{tr}}(\rho_x, \rho_y) = D_{\text{tr}}(\rho_y, \rho_x). \quad (14)$$

Closure under recovery maps ensures that the recovered representatives belong to the quotient sector. The  $O(\epsilon)$  statement follows from the triangle inequality for the chosen distinguishability metrics and the assumed  $\epsilon$ -closure of recovered distinguishabilities.  $\square$

**Remark 2** (What this theorem does and does not prove). *The theorem does not yet prove  $\mathbb{C}\mathbb{P}^{N-1}$ , the Born rule, or the full Hilbert reconstruction. It proves the first hard step: once directed record holonomy is operationally zero in a restricted access category, the induced reversible quotient has a symmetric distinguishability kernel. The symmetry arises from trace-distance symmetry in the restricted quantum-access category, not from a derivation of symmetric distinguishability from directed holonomy alone. The larger task — deriving Hilbert symmetry without importing a symmetric metric — is a separate reconstruction target.*

### THE HILBERT SECTOR AS A CONDITIONAL TARGET

**Criterion 1** (Hilbert-sector admissibility). *A zero-holonomy quotient is a Hilbert-sector candidate if it satisfies: finite capacity  $N$ , reciprocal distinguishability, basis equivalence, reversible dynamics preserving  $K_{\text{rev}}$ , composition stability, and no hidden record asymmetry.*

**Theorem 2** (Hilbert-sector theorem target). *A finite, reciprocal, composition-stable, zero-holonomy quotient with nontrivial reversible dynamics is represented by a finite-dimensional quantum sector*

$$\mathcal{X}_{\text{rev}} \simeq \mathbb{C}\mathbb{P}^{N-1}, \quad (15)$$

with

$$K_Q(\psi, \phi) = 1 - |\langle \psi | \phi \rangle|^2. \quad (16)$$

**Remark 3** (Relation to quantum reconstruction programs). *This theorem target should be connected to standard reconstruction strategies, including operational-probabilistic reconstructions, purification-based reconstructions, Jordan-algebraic routes, and distinguishability-based finite-capacity reconstructions. Q0's distinctive contribution is upstream: it does not assume symmetric distinguishability as primitive, but tries to explain when that symmetry appears.*

### BORN RULE PROGRAM: A COMPANION TARGET

The Born rule is deliberately not the central completed result of this paper. Q0 locates where the Born rule should enter: it should be the unique probability rule on the reversible quotient that prevents stable-record capacity arbitrage across bases.

**Criterion 2** (Record-capacity no-arbitrage). *A probability rule  $p(E|\psi)$  is admissible only if it satisfies: basis-neutral stable-record accounting, finite record-budget conservation, coarse-graining consistency, composition stability, and no basis-dependent creation of extra maintainable distinctions.*

**Model 1** (Illustrative deformation family). *Consider*

$$p_i^{(\alpha)} = \frac{|\psi_i|^\alpha}{\sum_j |\psi_j|^\alpha}. \quad (17)$$

*The Born rule is the case  $\alpha = 2$ . A restricted Q0B theorem may show that  $\alpha \neq 2$  generates basis-dependent stable-record inflation in finite sampling/record ledgers.*

**Theorem 3** (Born no-arbitrage theorem target). *Any probability rule satisfying finite record-capacity no-arbitrage, reversible-basis neutrality, coarse-graining consistency, and composition stability must be equivalent to the Born rule.*

**Remark 4** (Demotion rule). *If a non-Born rule satisfies no-arbitrage, basis neutrality, coarse-graining, and composition in a finite boundary-access setting, the Born-uniqueness claim is demoted. The core Q0 claim — zero holonomy produces the reversible symmetric sector — does not depend on completing this proof.*

### MACH-ZEHNDER STABLE-RECORD MODEL

This section gives the minimal operational model requested by Q0. The goal is not to derive a universal law of visibility suppression. The goal is to define a testable separation between ordinary overlap and stable accessible record holonomy.

**Model 2** (Mach-Zehnder / which-path record). *Let the path state after which-path coupling be*

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle|E_0\rangle + |1\rangle|E_1\rangle), \quad (18)$$

with ordinary coherence overlap

$$q = |\langle E_0 | E_1 \rangle|. \quad (19)$$

*Let a stable record channel retain a fraction  $r \in [0, 1]$  of path information with classical readout error  $e$ . Define the stable record information proxy*

$$I_{\text{rec}} = r [1 - h_2(e)], \quad (20)$$

where  $h_2$  is the binary entropy in bits. *Let the erasure irreversibility factor be*

$$\chi_{\text{erase}} = 1 - \exp(-\kappa_{\text{erase}}/C_{\text{erase}}). \quad (21)$$

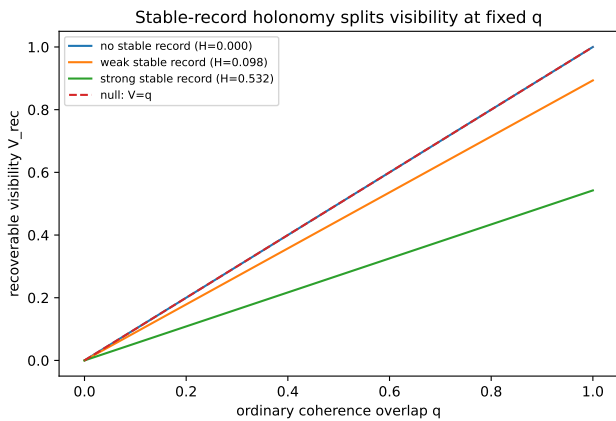


FIG. 1. Minimal Mach-Zehnder normal form. At fixed ordinary coherence overlap  $q$ , stable record holonomy suppresses recoverable visibility. The dashed curve is the null model  $V = q$ . The locked prediction is the independent record-holonomy dependence, not the exact exponential form.

The record-holonomy normal form is

$$H_{\text{rec}} = I_{\text{rec}} \chi_{\text{erase}}. \quad (22)$$

A minimal recoverable-visibility normal form is

$$V_{\text{rec}} = V_0 q \exp(-\lambda H_{\text{rec}}). \quad (23)$$

**Remark 5** (What is locked and what is not). The exponential form in Eq. (23) is phenomenological. Other monotone suppression functions are allowed. The locked Q0 prediction is the independent dependence on stable record holonomy after controlling ordinary overlap  $q$ , not the exact exponential curve.

**Protocol 1** (Stable-record versus mere-entanglement test). A minimal experimental protocol is:

1. Prepare a Mach-Zehnder or equivalent which-path interferometer.
2. Fix the ordinary environment overlap  $q = |\langle E_0 | E_1 \rangle|$  across experimental branches.
3. Vary stable record retention  $r$ , readout lifetime  $\tau_R$ , readout error  $e$ , or erasure cost  $C_{\text{erase}}$ .
4. Perform a quantum-eraser or reversal operation.
5. Measure recovered visibility  $V_{\text{rec}}$ .
6. Compare a null model  $V_{\text{rec}} = V_0 q$  against a record-holonomy model  $V_{\text{rec}} = F(q, H_{\text{rec}})$ .

The Q0 claim is demoted if stable accessible record structure has no independent predictive role once  $q$  and ordinary noise metrics are controlled.

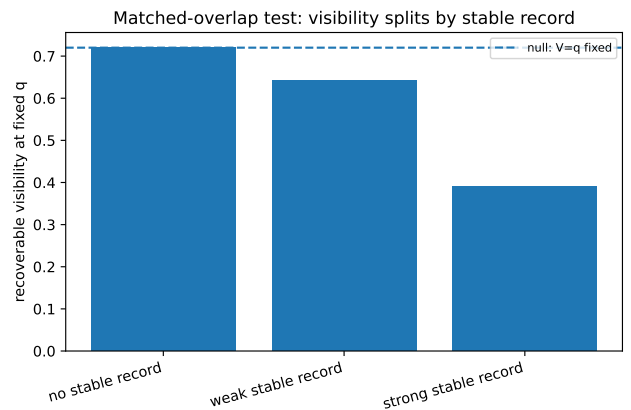


FIG. 2. Matched-overlap diagnostic. For fixed  $q = 0.72$ , visibility splits according to stable record holonomy. This is the most direct Q0 test: mere entanglement/overlap is held fixed while record accessibility changes.

## QEC DIRECTED-LEAKAGE MODEL

Q2 treats fault-tolerant quantum computation as protected finite distinction maintenance. Q0 adds the upstream holonomy language: leakage from the computational subspace is a damaging error source that standard QEC does not remove without leakage-reduction mechanisms [11]. Q0 packages this as directed leakage holonomy into uncorrectable boundary sectors.

**Definition 12** (Directed leakage holonomy). Let  $Q_L$  be a logical quantum distinction,  $S_{\text{syn}}$  a syndrome record, and  $E_{\text{uncorr}}$  an uncorrectable environment/boundary sector. Define

$$H_{\text{leak}} = I(Q_L; E_{\text{uncorr}} | S_{\text{syn}}). \quad (24)$$

This is the information about the logical quotient that remains outside the correctable syndrome-conditioned sector.

**Model 3** (QEC leakage normal form). For a code of distance  $d$  and physical error proxy  $p$ , let

$$P_L(p, d, H_{\text{leak}}) \approx A \left( \frac{p}{p_{\text{th}}} \right)^{(d+1)/2} + \gamma H_{\text{leak}}. \quad (25)$$

This is a toy proxy, not a threshold theorem. It says that logical failure may split at matched  $p$  if directed leakage differs.

**Protocol 2** (Directed-leakage QEC test). Compare noise channels matched in average physical error, syndrome rate, and ordinary fidelity but differing in uncorrectable leakage information  $H_{\text{leak}}$ . Q0 predicts that logical failure should track  $H_{\text{leak}}$  after standard metrics are controlled. The bridge is demoted if  $H_{\text{leak}}$  adds no predictive power beyond standard noise descriptors.

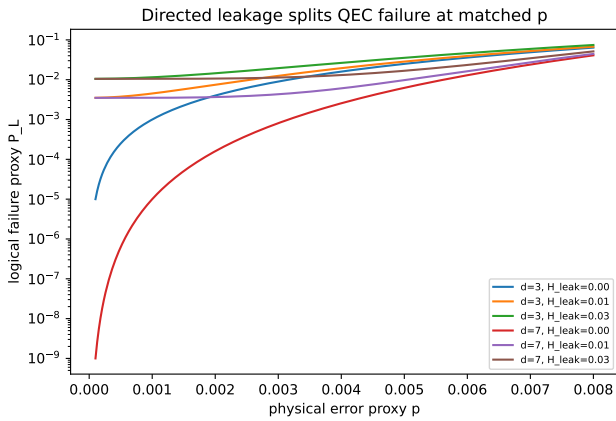


FIG. 3. QEC directed-leakage normal form. Logical failure is modeled as ordinary code suppression plus an independent uncorrectable leakage term. The point is not the exact exponent or coefficient, but the predicted split at matched physical error when directed leakage differs.

## MEASUREMENT, DECOHERENCE, AND FACTS

### Measurement as nonzero record holonomy

Q0 does not introduce fundamental collapse. It defines measurement as the stabilization of directed access asymmetry into a boundary-accessible record.

**Definition 13** (Record promotion). *A record is promoted when*

$$I(S; R_B) \geq I_c, \quad \tau_R \geq \tau_{\text{read}}, \quad (26)$$

where  $I(S; R_B)$  is system-record mutual information,  $I_c$  is the promotion threshold,  $\tau_R$  the record lifetime, and  $\tau_{\text{read}}$  the readout timescale.

**Proposition 1** (Measurement-holonomy normal form). *A quantum interaction belongs to the reversible quotient when  $\mathcal{H}_\partial = 0$  and no stable record is promoted. It belongs to the measurement sector when  $\mathcal{H}_\partial \neq 0$  and the record-promotion condition is satisfied.*

### Decoherence as ledger export

Decoherence is often described as entanglement with an environment. Q0 refines the operational criterion: irreversibility tracks stable boundary-accessible record structure, not entanglement alone [10]. Entanglement can be reversible; stable records are costly to erase.

## RELATION TO EXISTING QUANTUM RECONSTRUCTIONS

Q0 should be compared with quantum reconstruction programs that derive or motivate Hilbert space, purification, the Born rule, or tensor-product composition from operational principles. The FDS distinction is upstream: Q0 does not begin with symmetric distinguishability as primitive. It first defines directed finite boundary access and asks when the zero-holonomy quotient becomes symmetric.

A useful comparison is with distinguishability-based finite-capacity reconstructions and generalized probabilistic theories [4–7]. Such programs often begin with a graded equality or symmetric distinguishability kernel and impose strong closure requirements. Q0 instead asks how a symmetric kernel becomes available from asymmetric access in the first place. Therefore Q0 is compatible with these reconstructions as a second-stage theorem, but does not inherit their strongest closure assumptions as primitive.

## PREDICTION LOCK AND DEMOTION LADDER

**Criterion 3** (Prediction A: stable record versus mere entanglement). *Recoverable interference should depend on stable accessible record holonomy after controlling ordinary overlap/entanglement. Failure to observe any independent stable-record effect demotes the record-holonomy branch.*

**Criterion 4** (Prediction B: access-holonomy reversibility). *If  $\mathcal{H}_\partial = 0$ , full interference recovery should be possible in principle. If  $\mathcal{H}_\partial \neq 0$ , recovery requires explicit record erasure, boundary reversal, or maintenance cost. If measured reversibility is independent of any operational holonomy quantity, the holonomy formulation is demoted.*

**Criterion 5** (Prediction C: directed-leakage QEC). *Logical failure should depend on directed leakage holonomy after controlling average physical error and ordinary noise metrics. If it does not, the Q0/Q2 bridge is demoted.*

**Criterion 6** (Optional frontier prediction: finite-capacity floor). *If finite capacity is fundamental rather than merely operational, sufficiently precise experiments may encounter a finite distinguishability resolution floor. This is not part of the core Q0 validation unless a concrete  $N_{\text{eff}}$  and scaling law are specified.*

## DISCUSSION

The main improvement of this version is that  $\mathcal{H}_\partial$  is no longer merely a metaphor. In a restricted access cate-

gory, zero record holonomy makes the reversible quotient well-defined; in the restricted quantum-access category the recovered-state kernel is symmetric. In operational models, nonzero record holonomy suppresses recoverable visibility beyond ordinary overlap, while directed leakage holonomy splits QEC logical failure beyond average physical error. These are modest claims compared with a complete derivation of quantum mechanics, but they are harder claims: they can be formalized, simulated, and experimentally challenged.

The appropriate next steps are therefore narrow. First, strengthen the restricted theorem by expanding the access category and proving uniqueness of the reversible quotient under weaker assumptions. Second, sharpen the Mach-Zehnder protocol into an experimental design with controls, estimators, and null models. Third, develop the QEC leakage model for concrete channels: Pauli-stochastic noise, coherent leakage, measurement-record leakage, and flagged erasure-like leakage. Fourth, move the Born-rule no-arbitrage program into a dedicated technical appendix or companion paper.

## CONCLUSION

FDS-Q0 locates standard finite-dimensional quantum mechanics as the zero-holonomy sector of asymmetric finite boundary access. The closed Hilbert formalism is not denied; it is located. It is the reversible quotient obtained when directed access asymmetry, stable records, irreversible update, and uncorrectable leakage are removed. Measurement, decoherence, and QEC overhead are not primitive exceptions to quantum theory; they are nonzero boundary-access holonomy sectors.

The compressed thesis is

$$\boxed{\begin{aligned} \mathcal{H}_\partial = 0 &\Rightarrow \text{Hilbert QM,} \\ \mathcal{H}_\partial \neq 0 &\Rightarrow \text{record/decoherence/maintenance sectors.} \end{aligned}} \quad (27)$$

This paper gives a restricted theorem, operational normal forms, and toy models. It does not yet complete the Hilbert-sector or Born-rule reconstruction. Its immediate scientific task is sharper: make boundary-access holonomy a calculable and testable quantity.

Q0 supplies an upstream quantum-access layer for the FDS quantum sequence.

## Notation Summary

TABLE III. Notation used in Q0.

Symbol	Meaning
$\mathcal{Q}$	finite boundary-access system
$\partial X$	access boundary
$\mathcal{I}$	boundary instrument/interaction class
$\mathcal{A}_\partial^{\mathcal{I}}(x \rightarrow y)$	directed access distinguishability
$\Delta_\partial^{\mathcal{I}}$	access asymmetry
$\mathcal{H}_\partial$	boundary-access holonomy
$K_{\text{rev}}$	reversible quotient kernel
$H_{\text{rec}}$	stable record holonomy proxy
$H_{\text{leak}}$	directed leakage holonomy proxy
$q$	ordinary environment overlap
$V_{\text{rec}}$	recoverable visibility
$Q_L$	logical quantum distinction
$S_{\text{syn}}$	syndrome record

## Model Reproducibility

The accompanying script `FDS_Q0_access_holonomy_model.py` produces the Mach-Zehnder and QEC figures used in this draft. The script writes CSV files for all plotted curves. The formulas are normal forms; they are intended to expose discriminating variables, not to claim universal microscopic dynamics.

## AI Assistance Disclosure

Drafting, organization, and code scaffolding for this manuscript were prepared with AI assistance under the author's direction. The theoretical claims, scope boundaries, and final responsibility remain with the author.

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